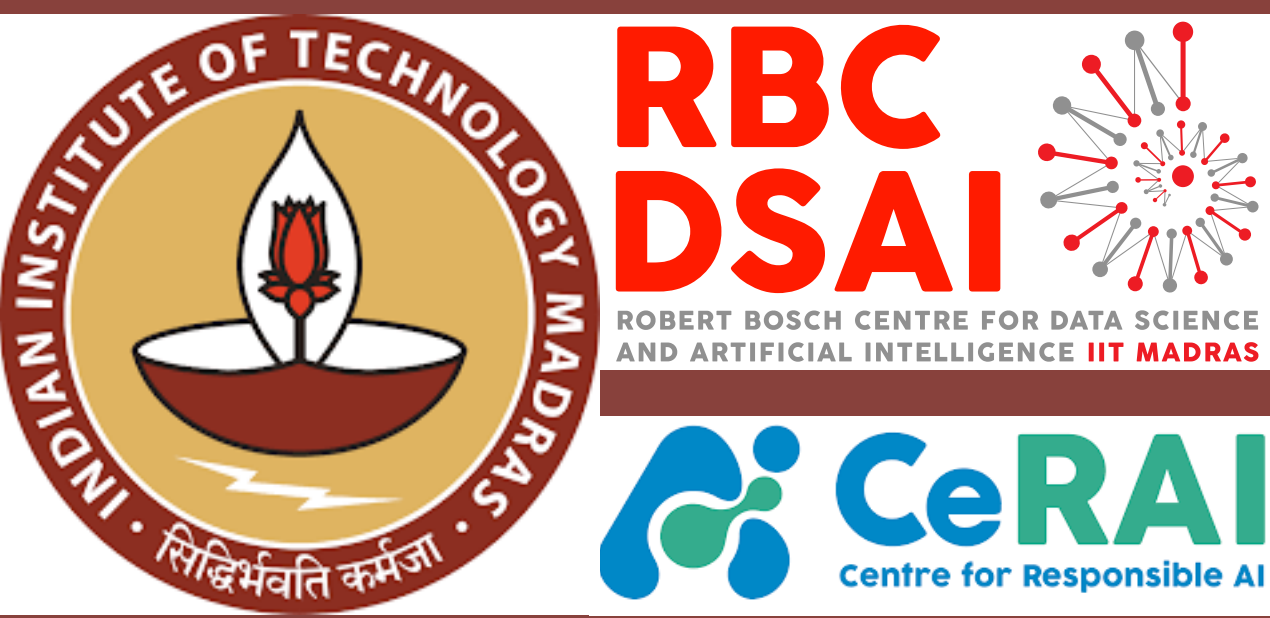


# Robust Aggregation for Federated Learning

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IEEE Transactions on Signal Processing (2022)

***Krishna Pillutla***  
IIT Madras





# Team

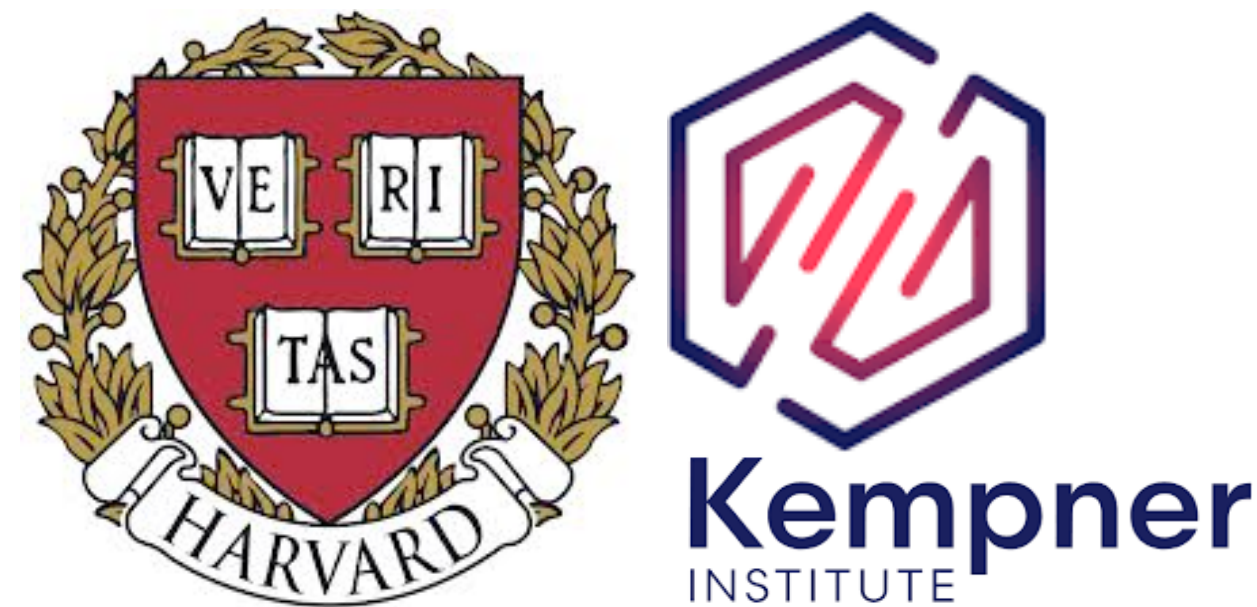
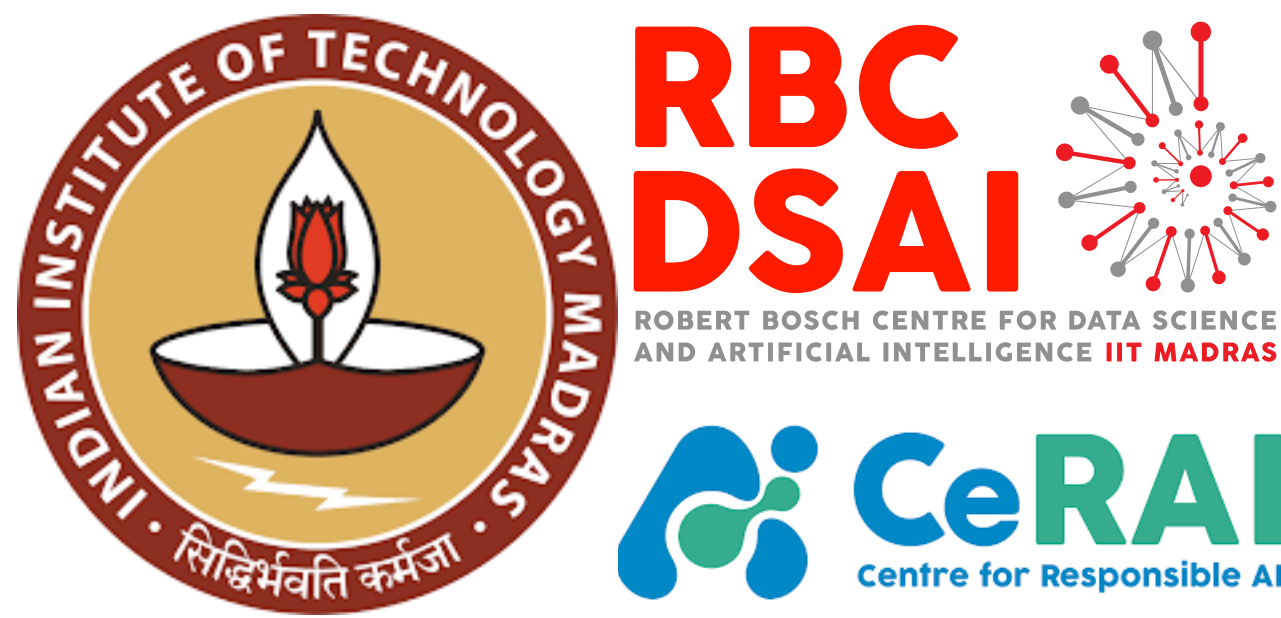
Krishna Pillutla



Sham Kakade



Zaid Harchaoui









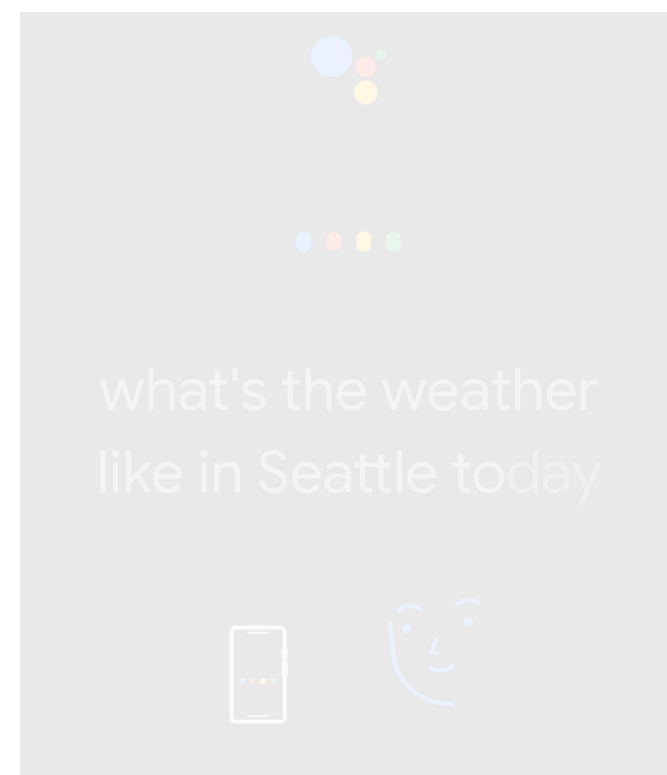
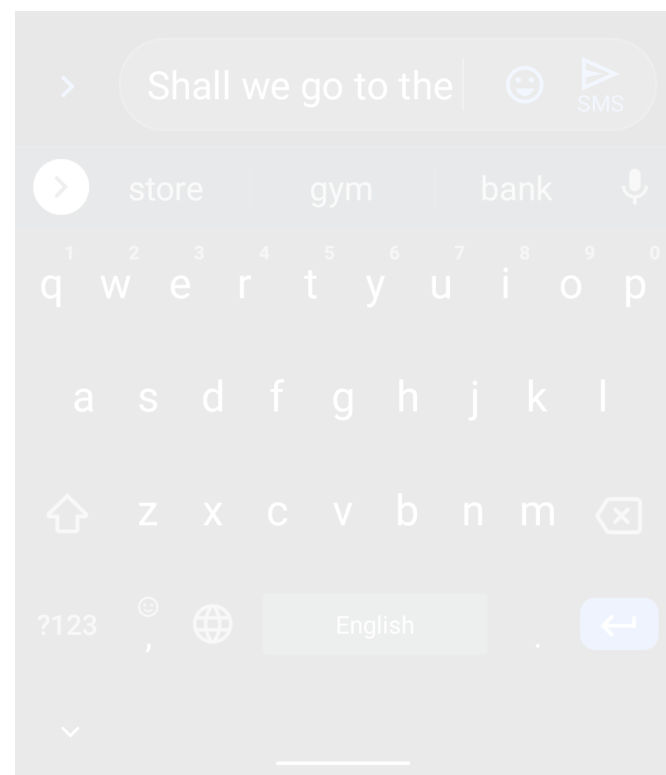
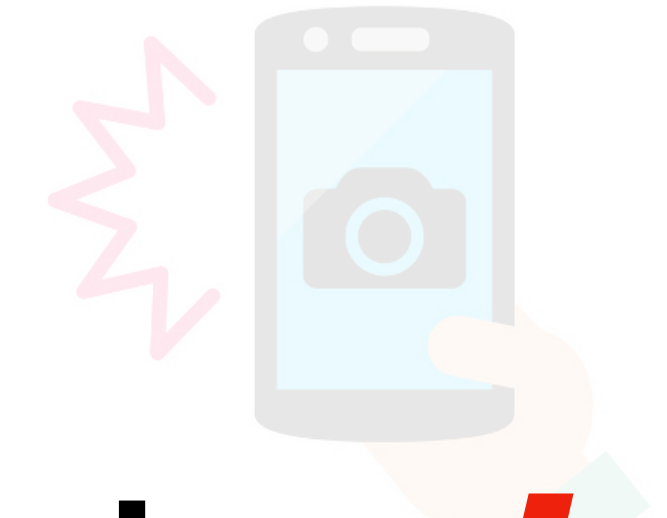
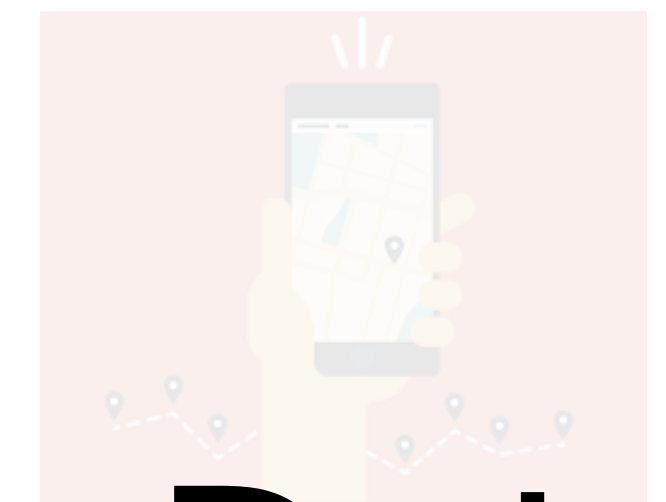


Image Credit: Robotics Business Review



Data is ***decentralized*** and ***private***

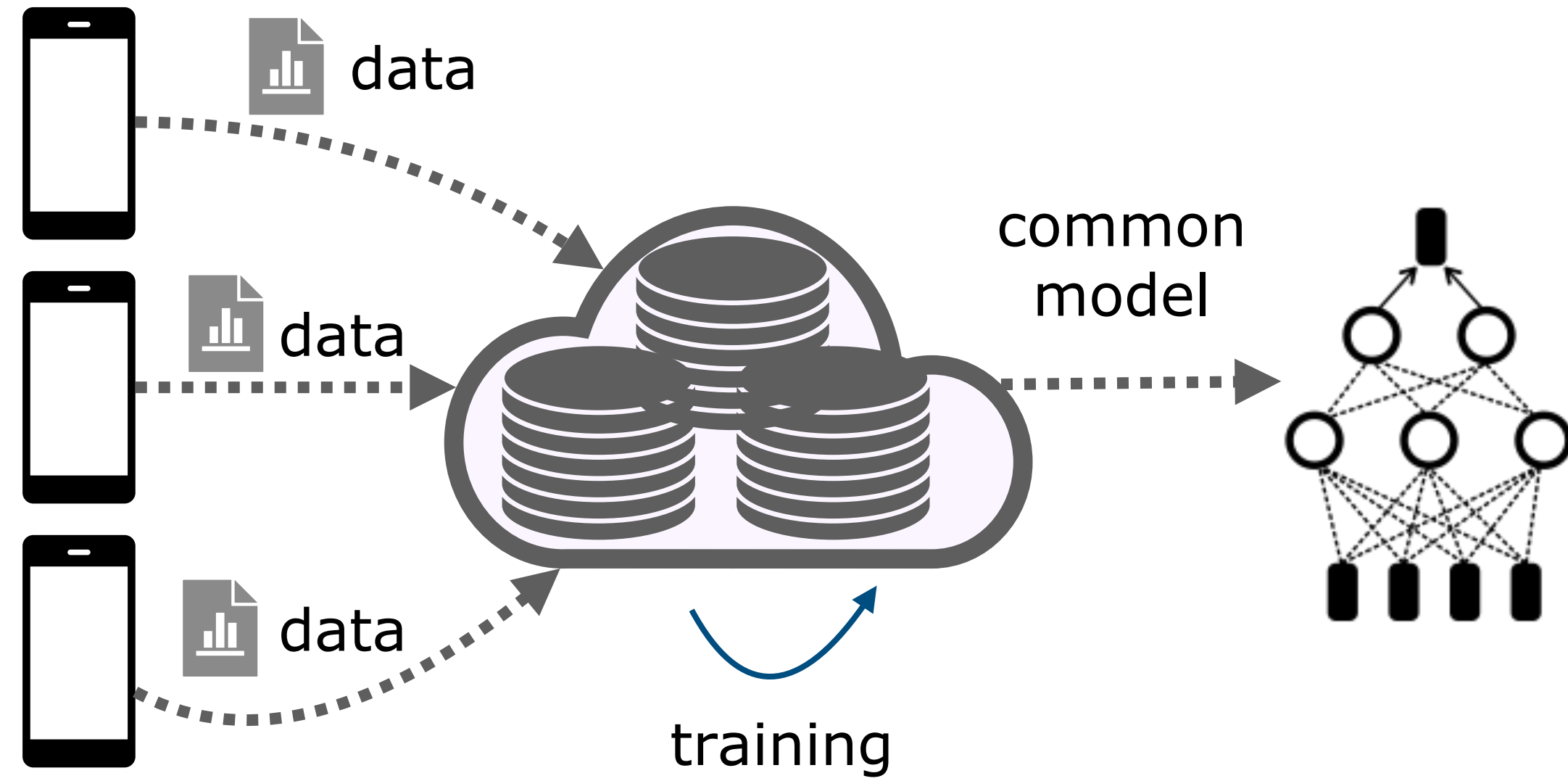


Rieke et al. NPJ Digit. Med. (2020)

Image Credit: Wellcome

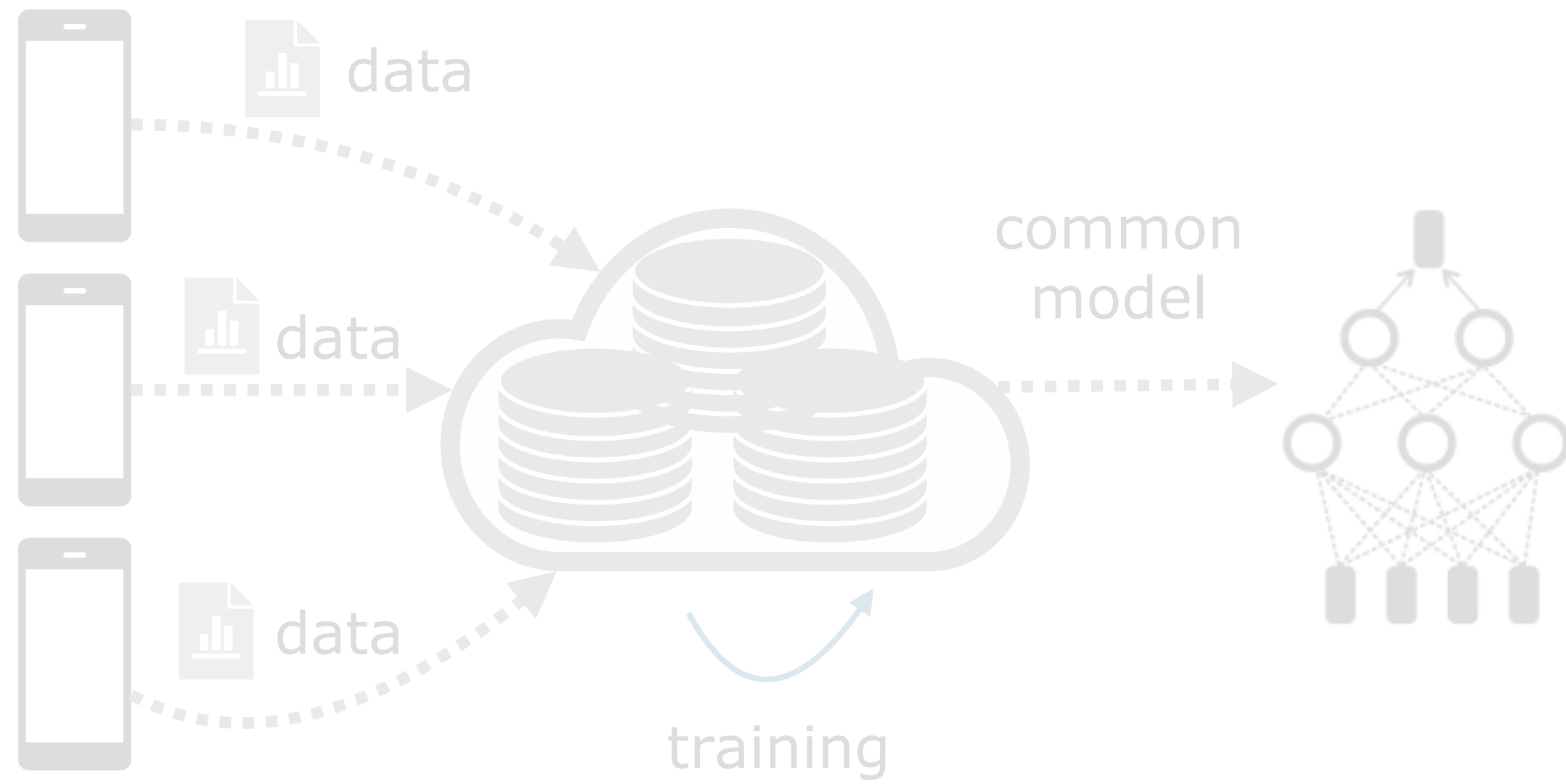


# Datacenter

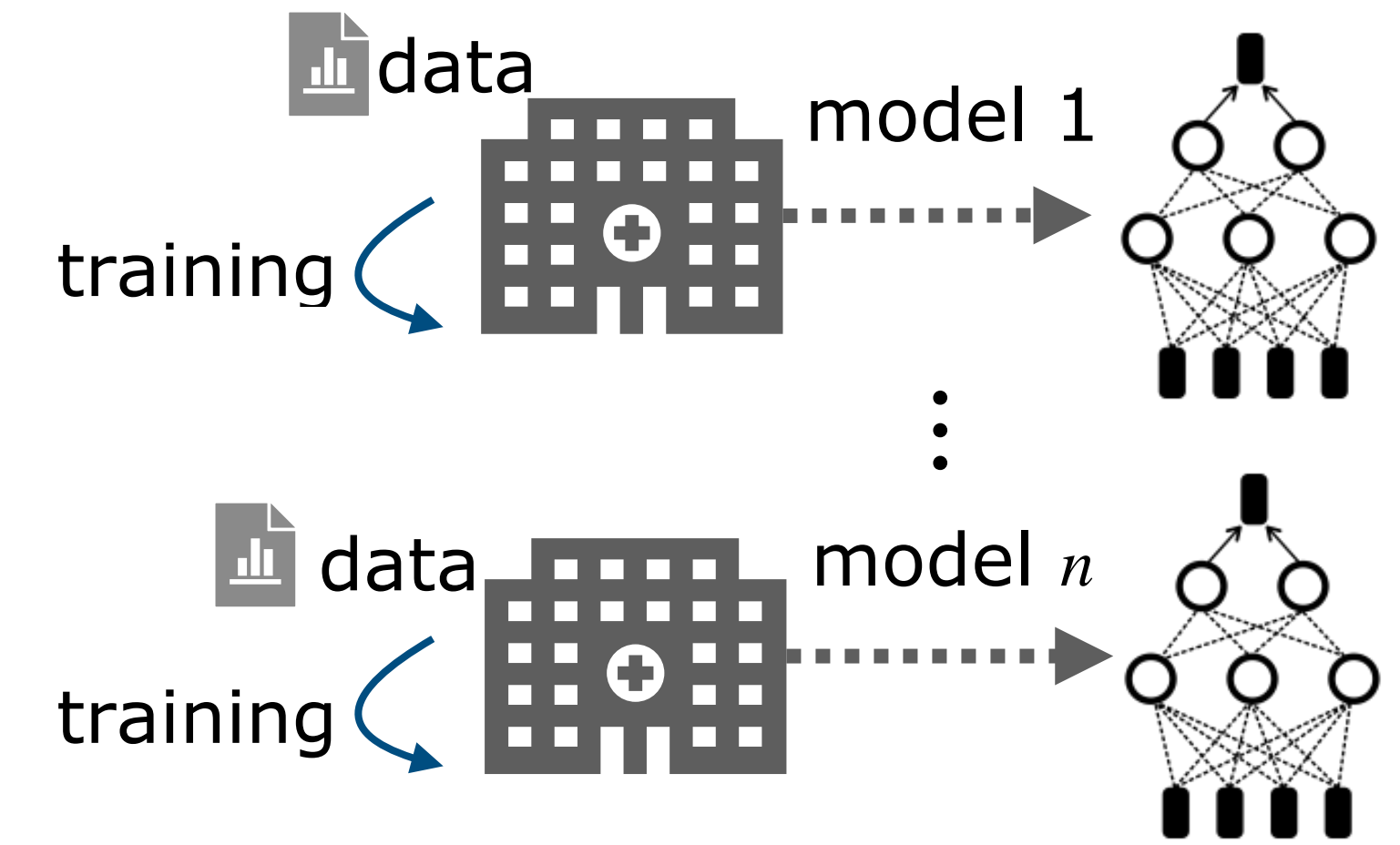




# Datacenter

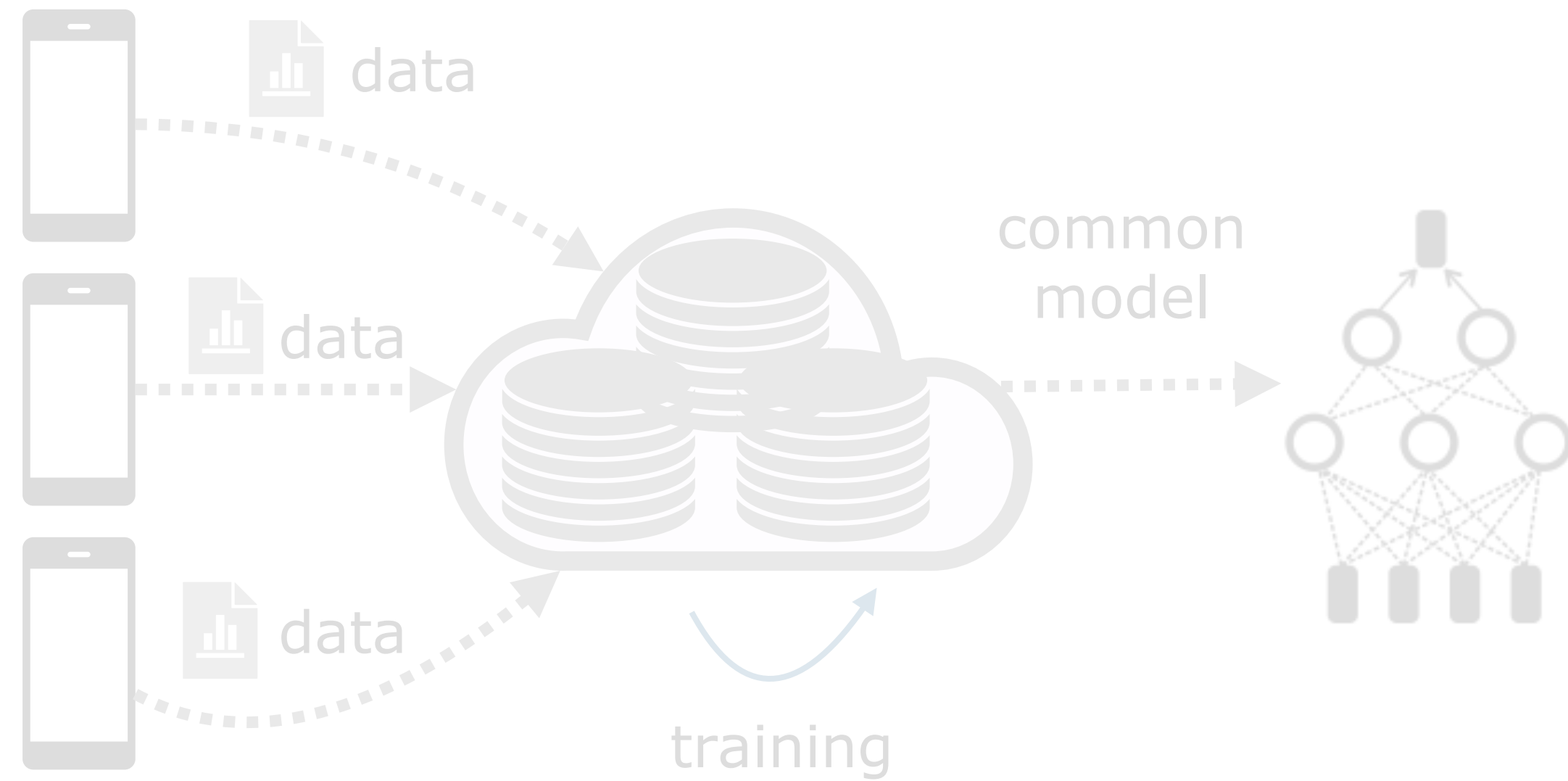


# Non-collaborative

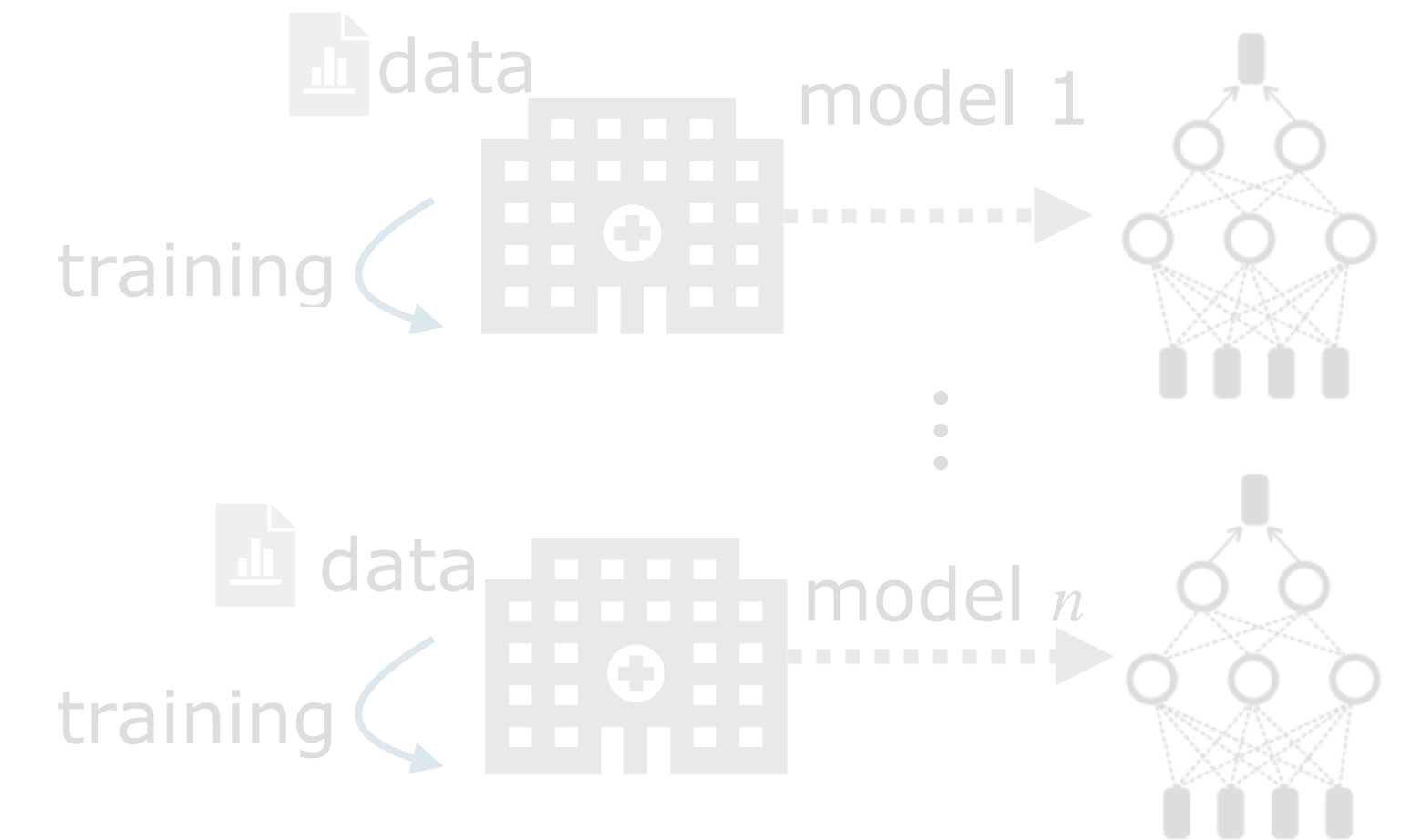




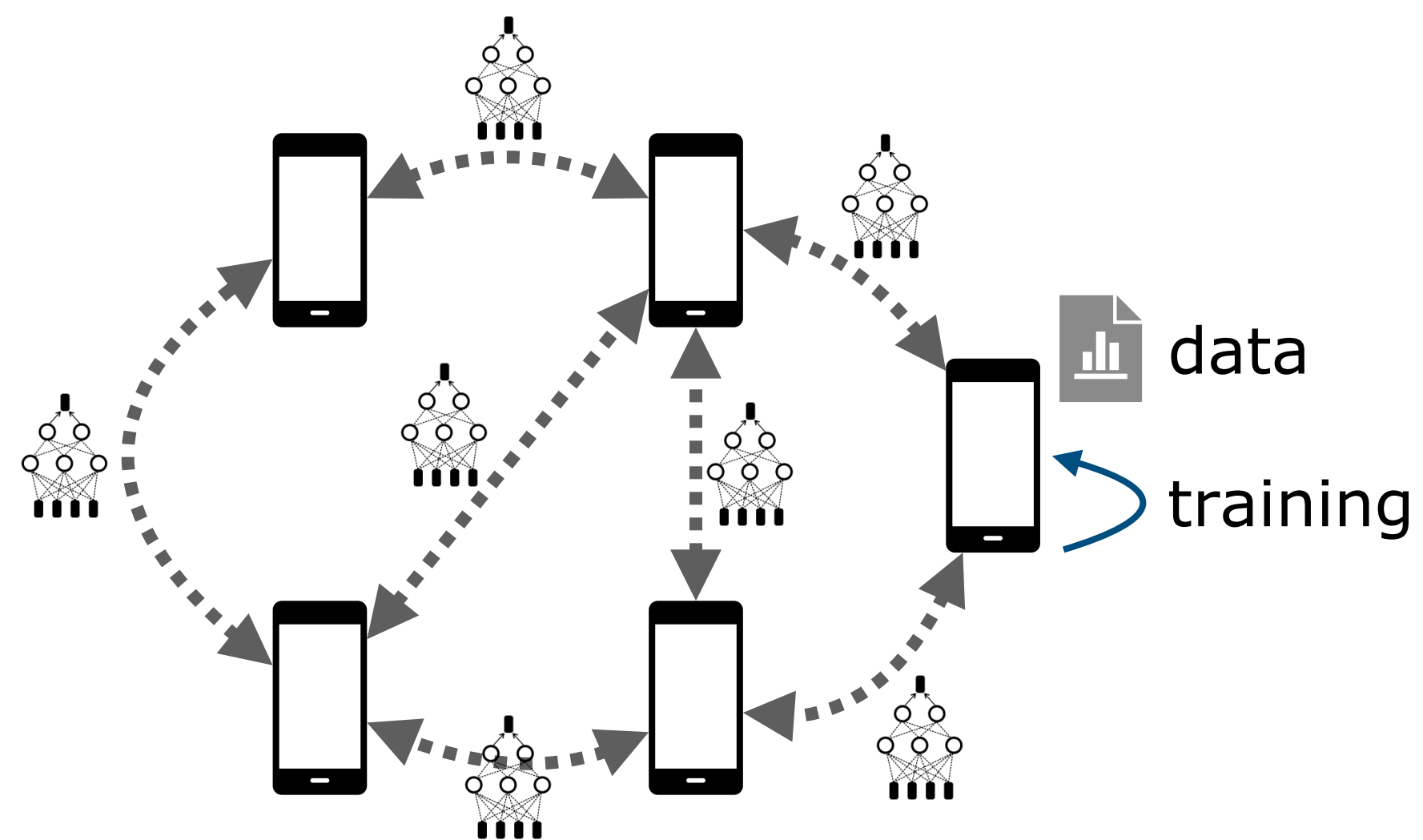
# Datacenter



# Non-collaborative



# Peer-to-peer





# Advances and Open Problems in Federated Learning

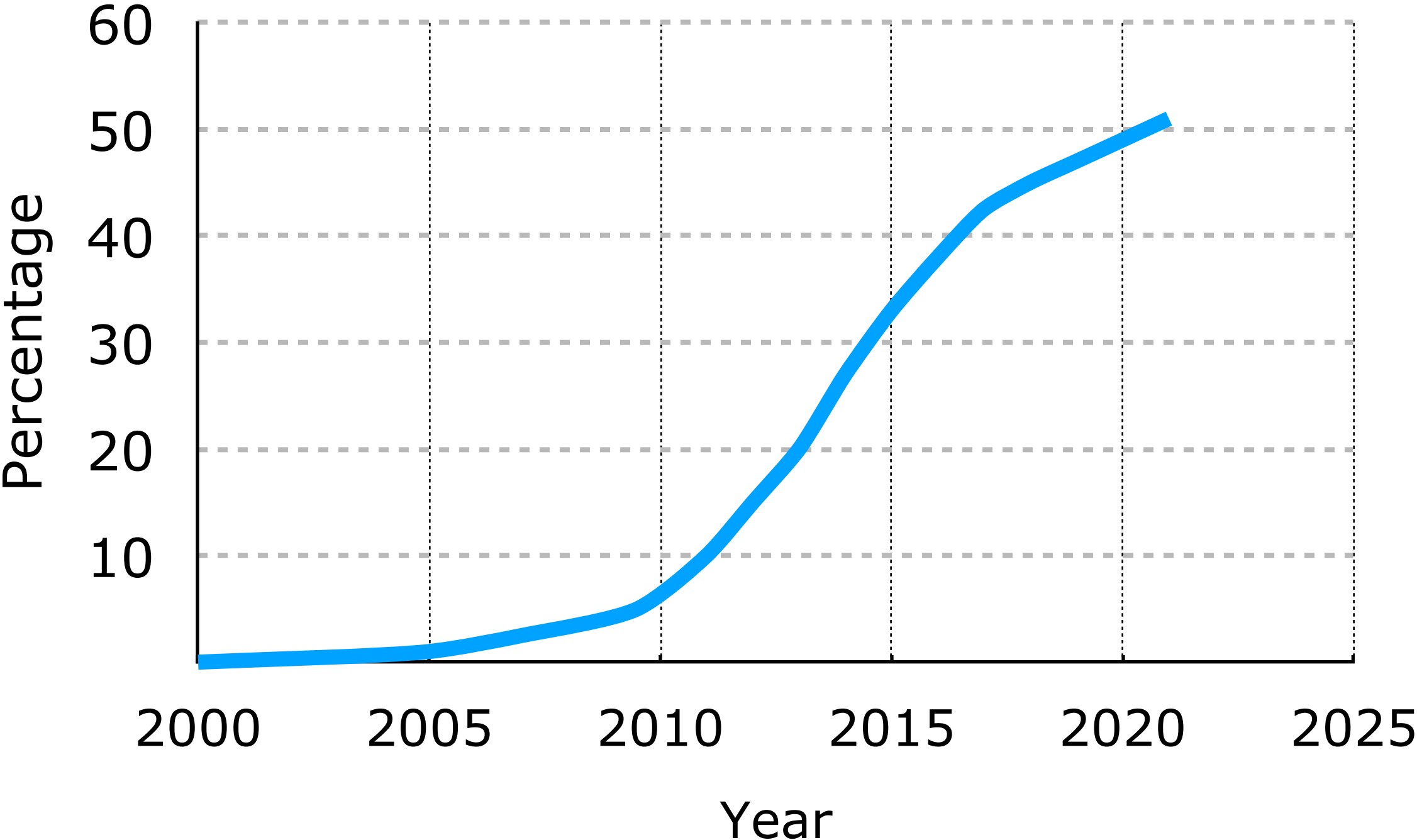
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**Peter Kairouz**  
Google Research  
Kairouz@google.com

**H. Brendan McMahan**  
Google Research  
*et al.*

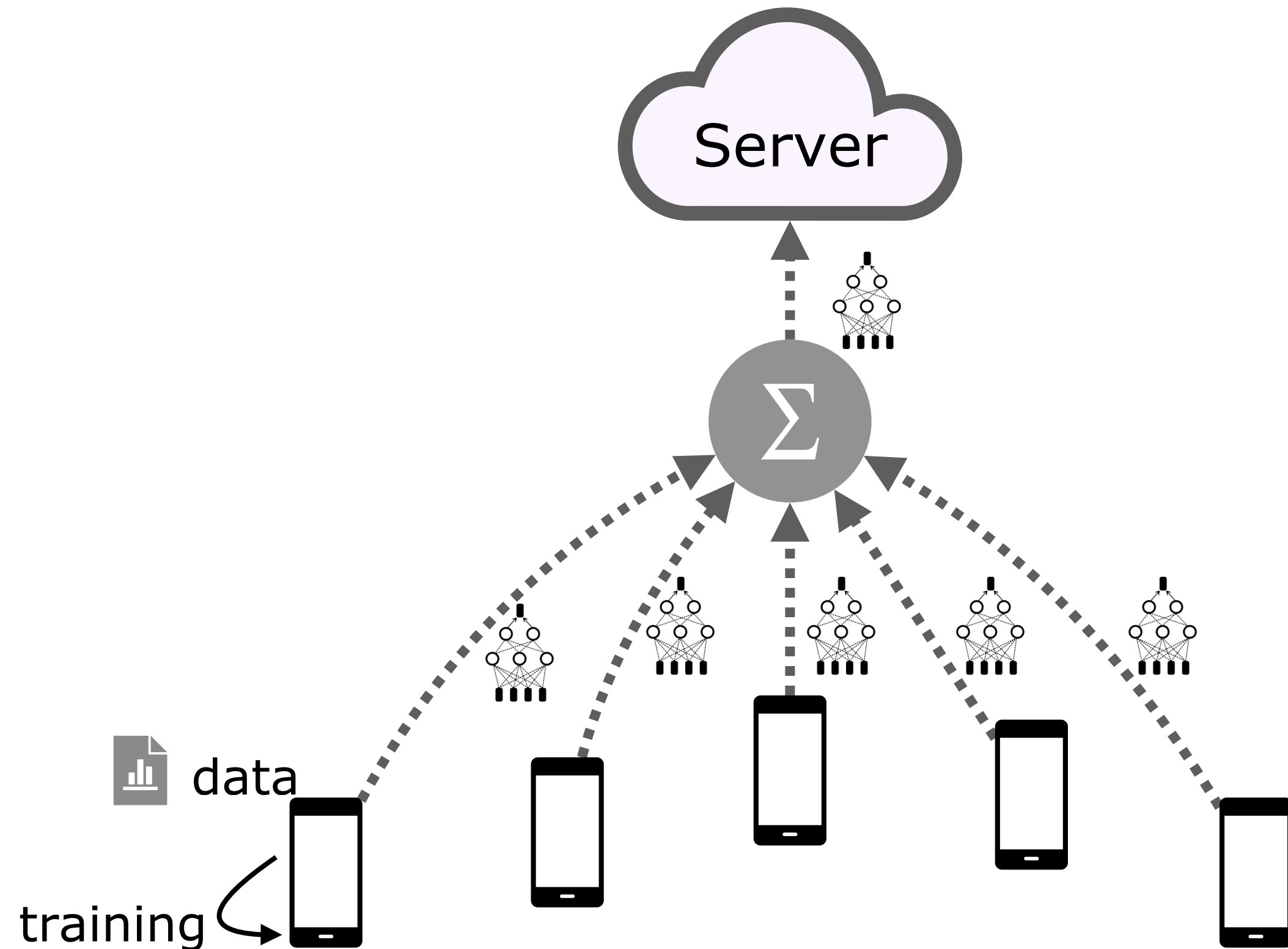


Percentage of world population with a smartphone (Data: Business Wire)

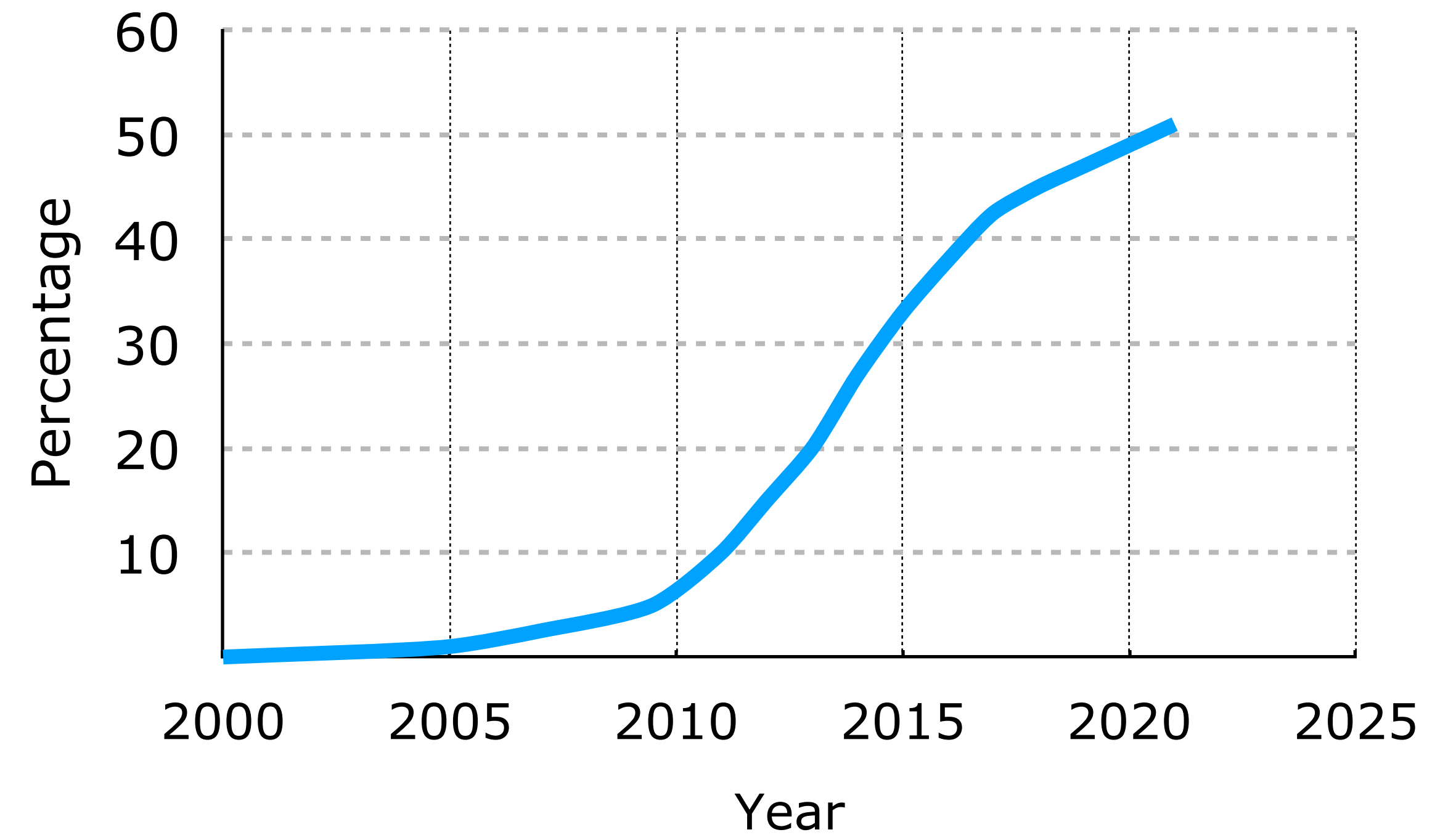




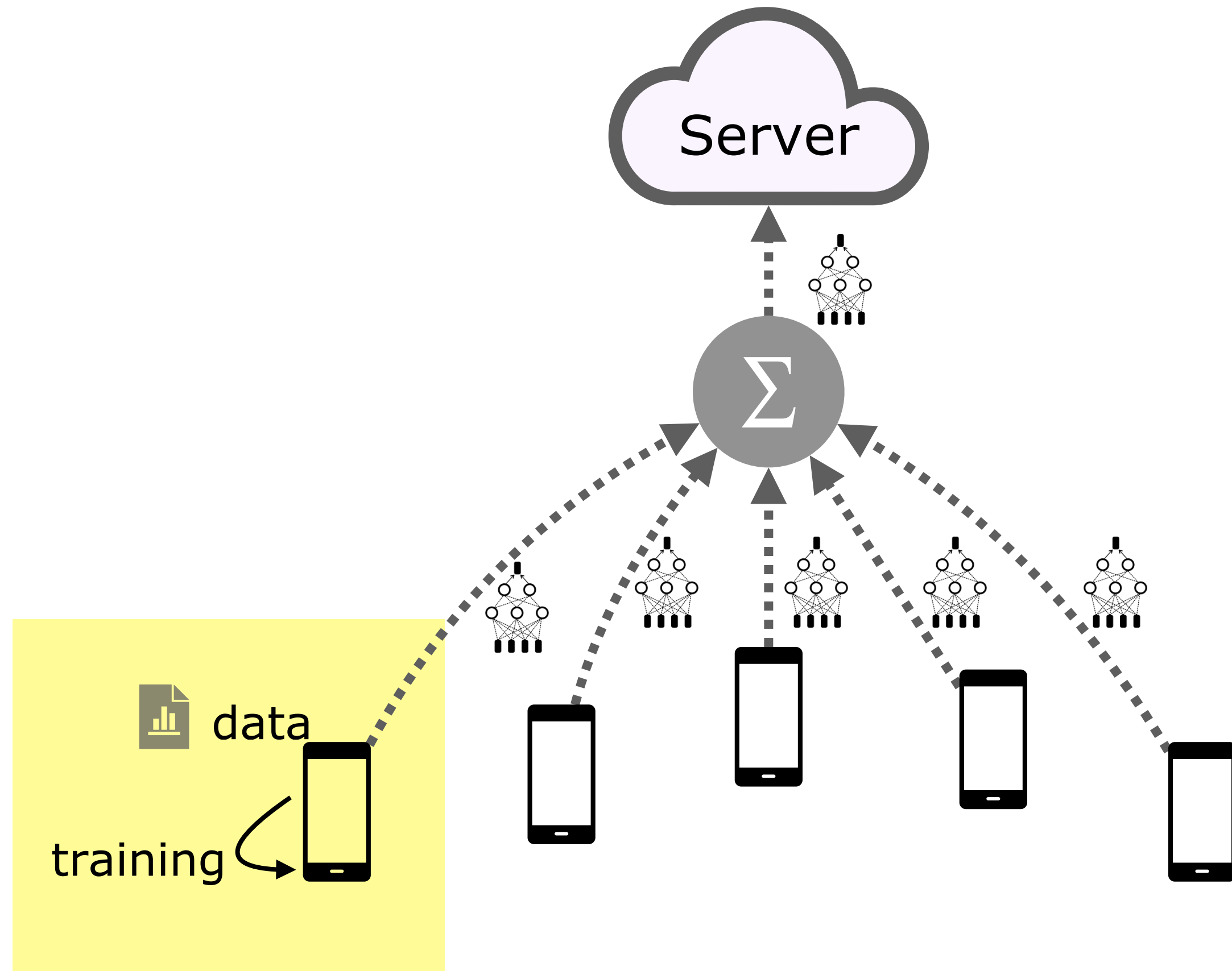
# Federated Learning



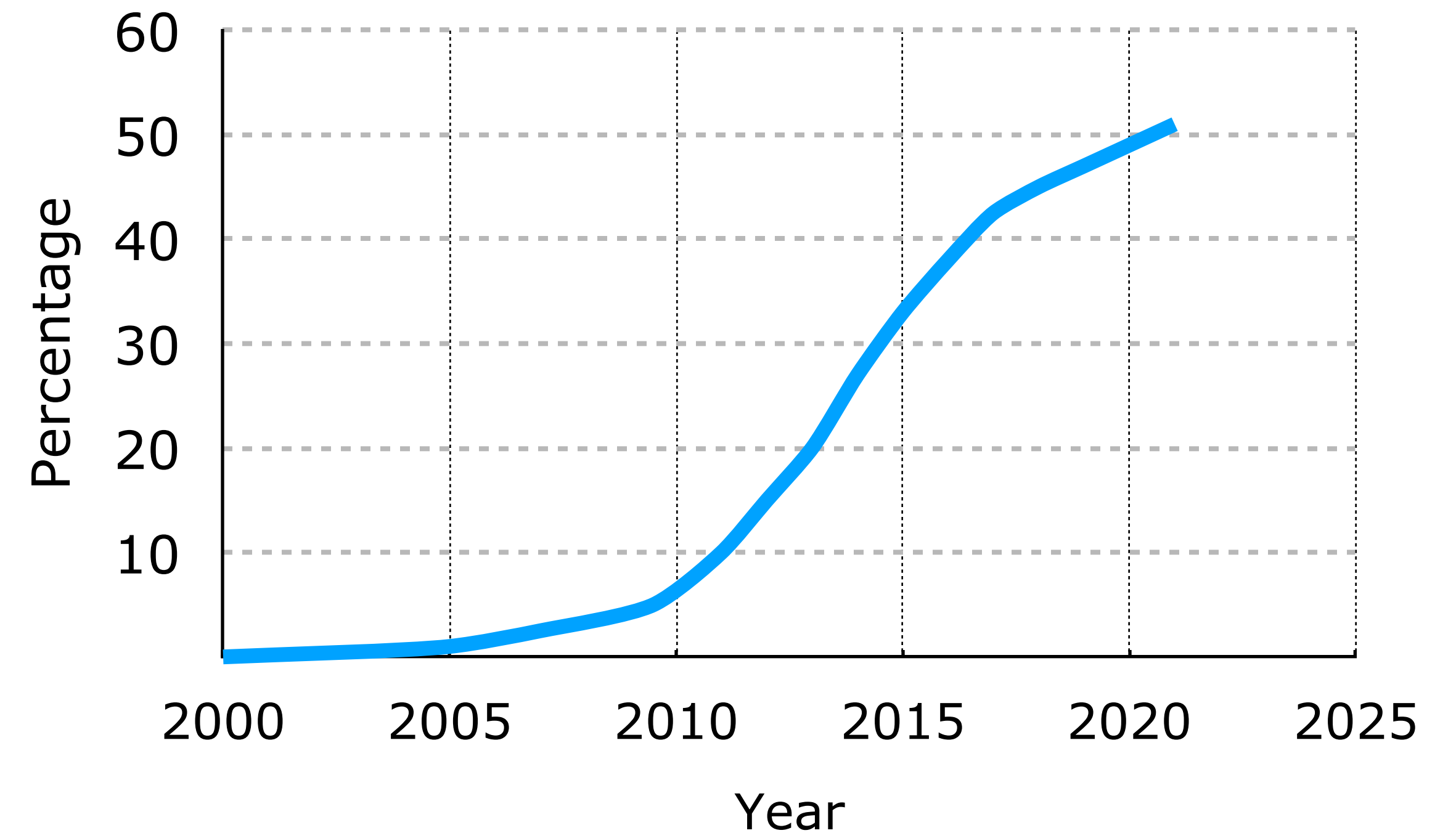
Percentage of world population with a smartphone (Data: Business Wire)



# Federated Learning

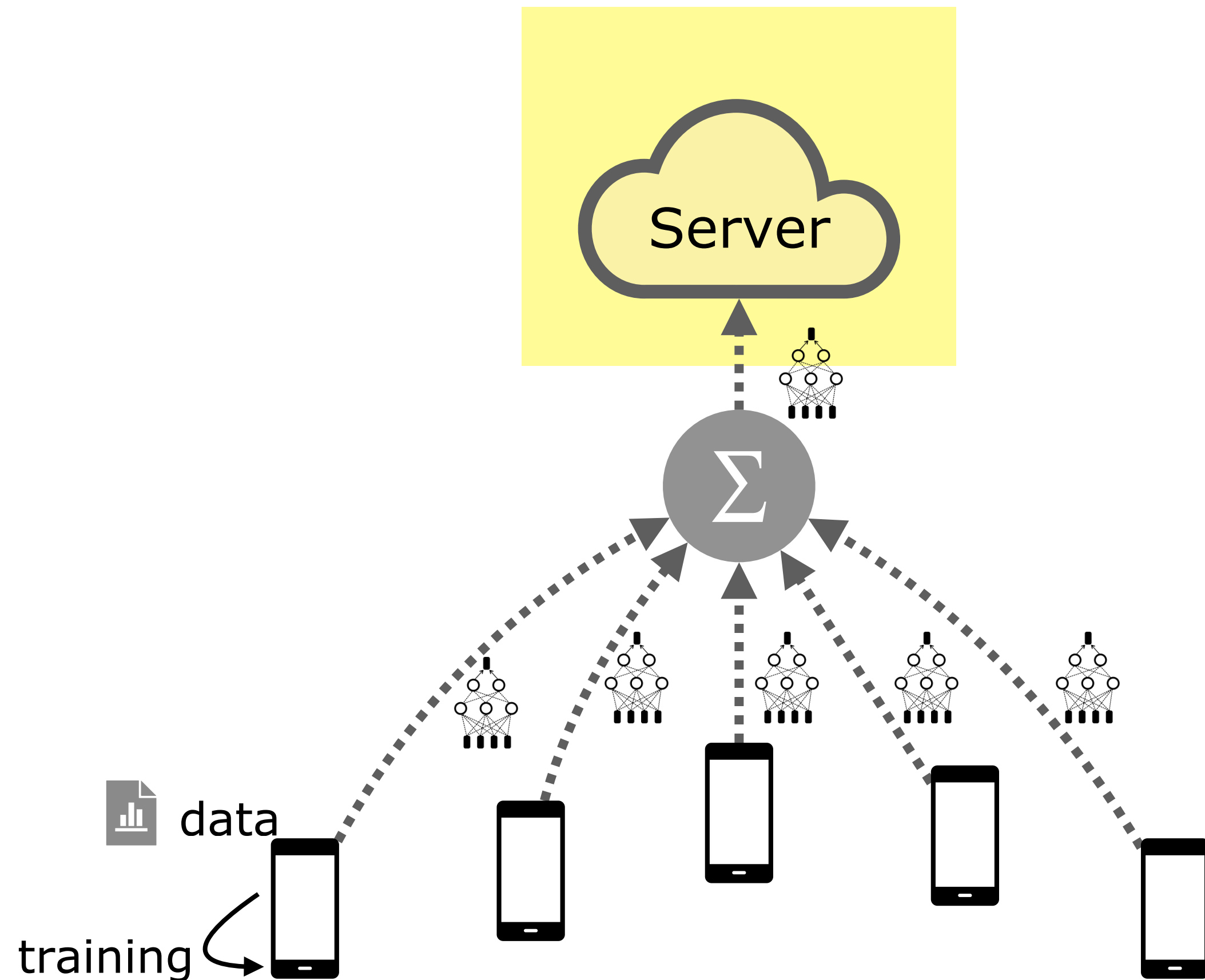


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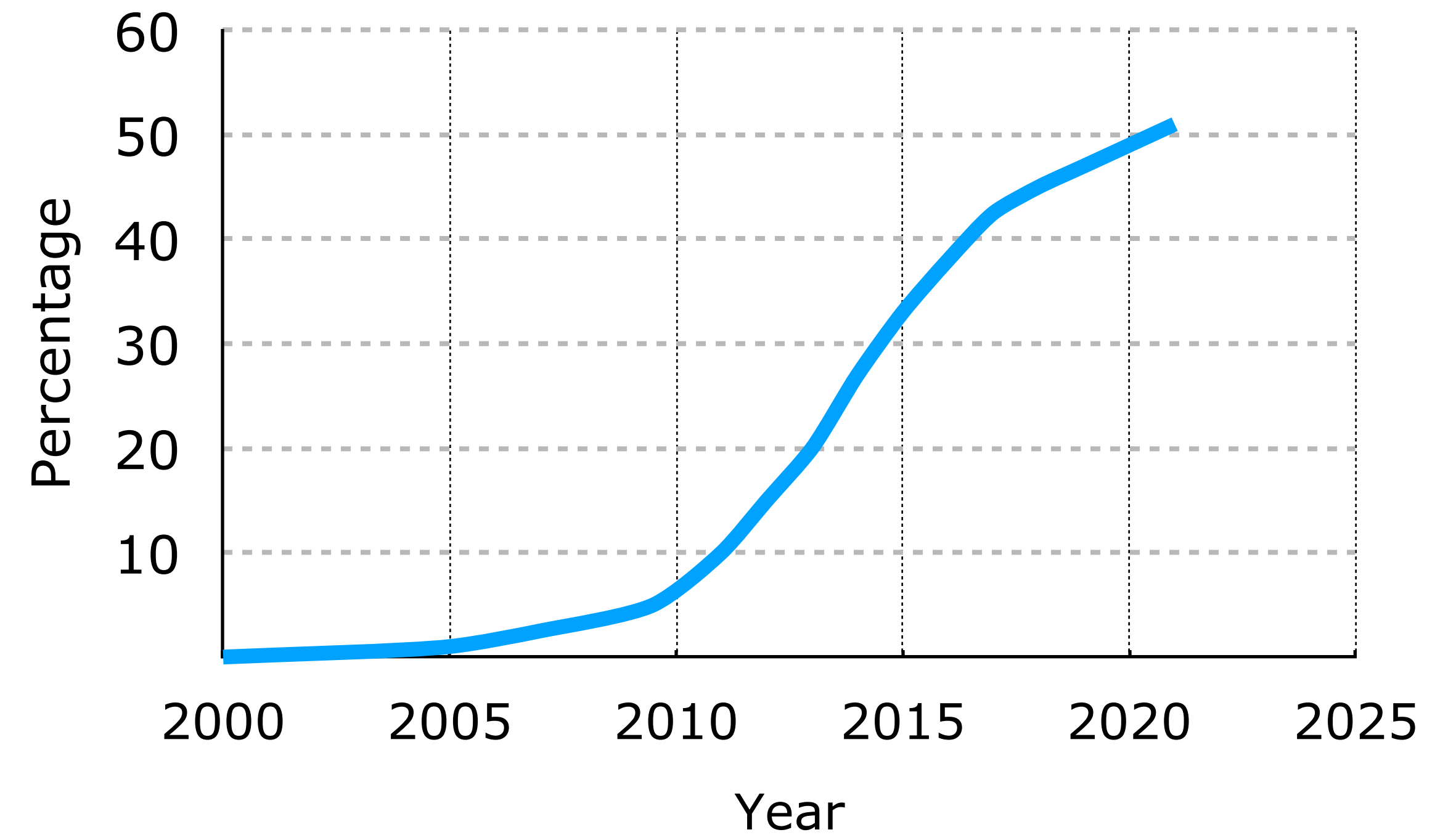




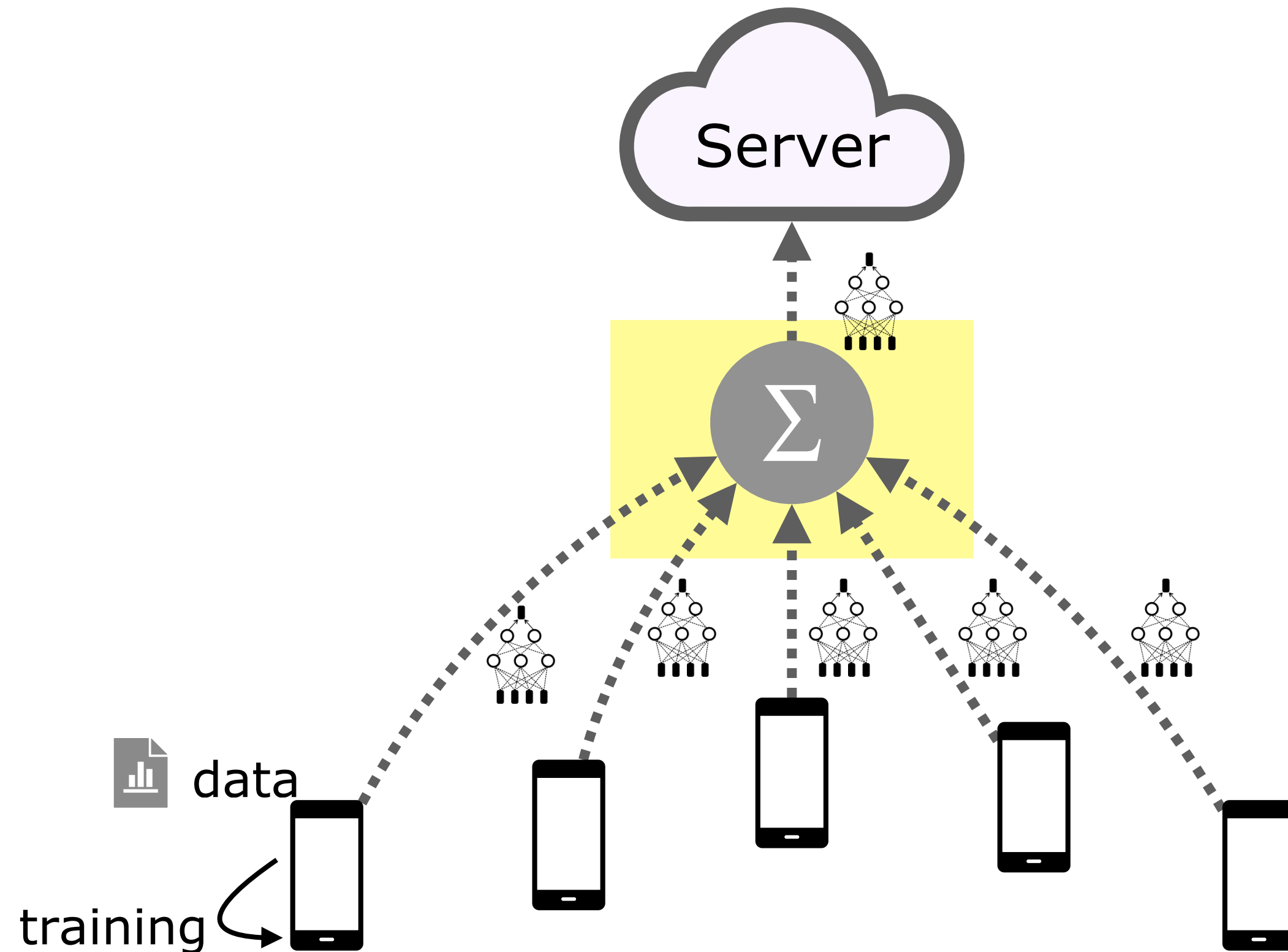
# Federated Learning



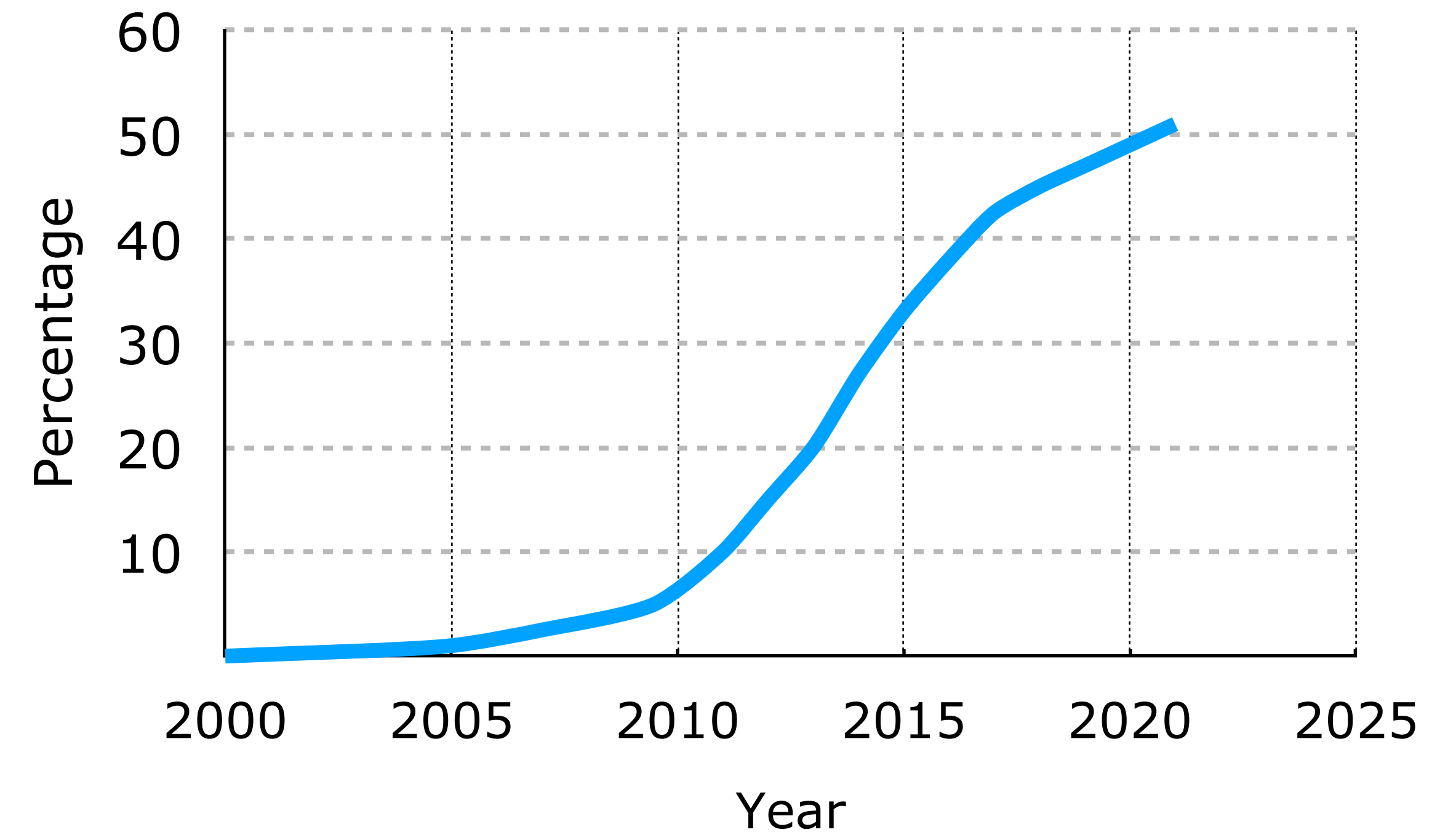
Percentage of world population with a smartphone (Data: Business Wire)



# Federated Learning

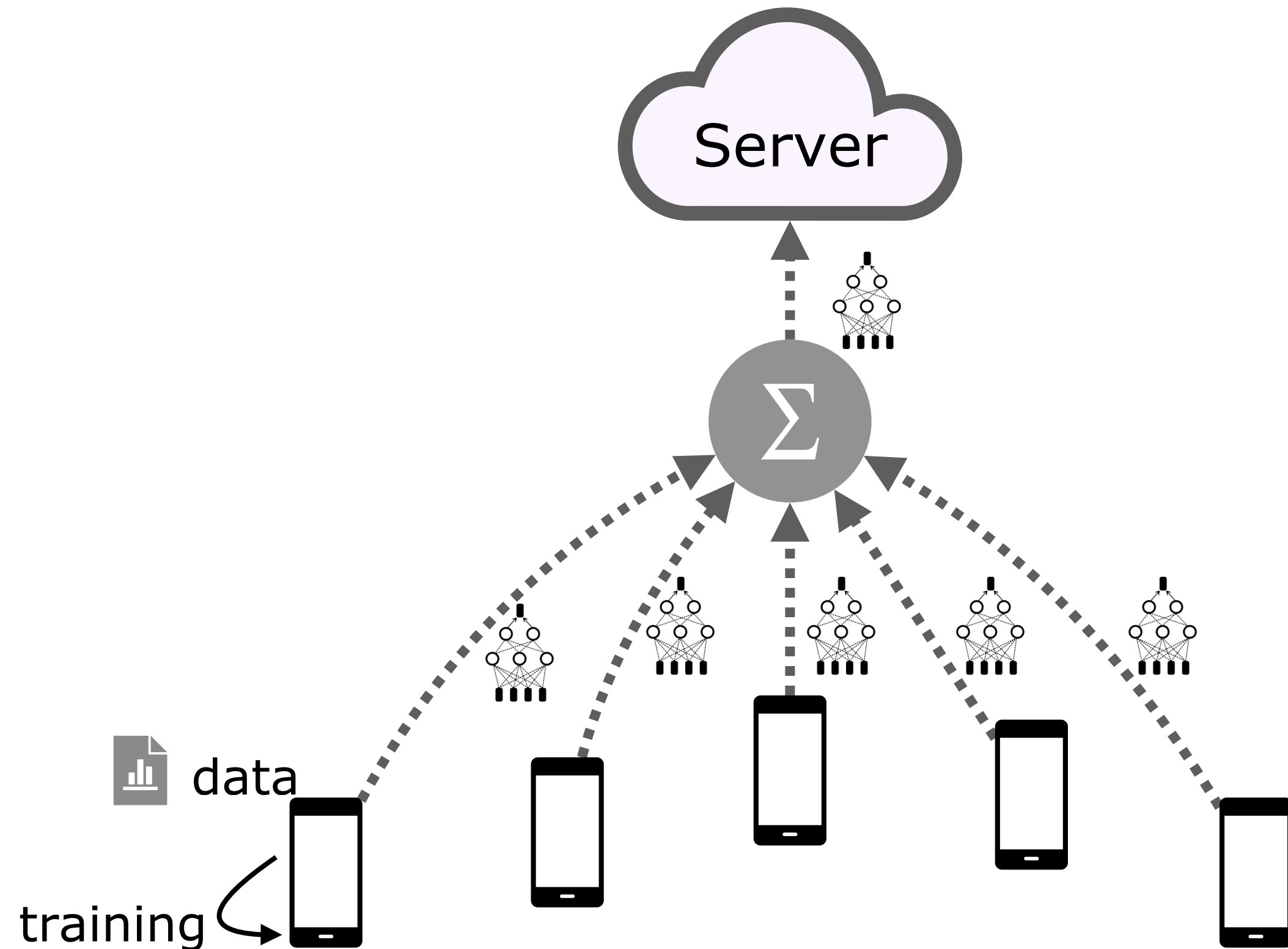


Percentage of world population with a smartphone (Data: Business Wire)

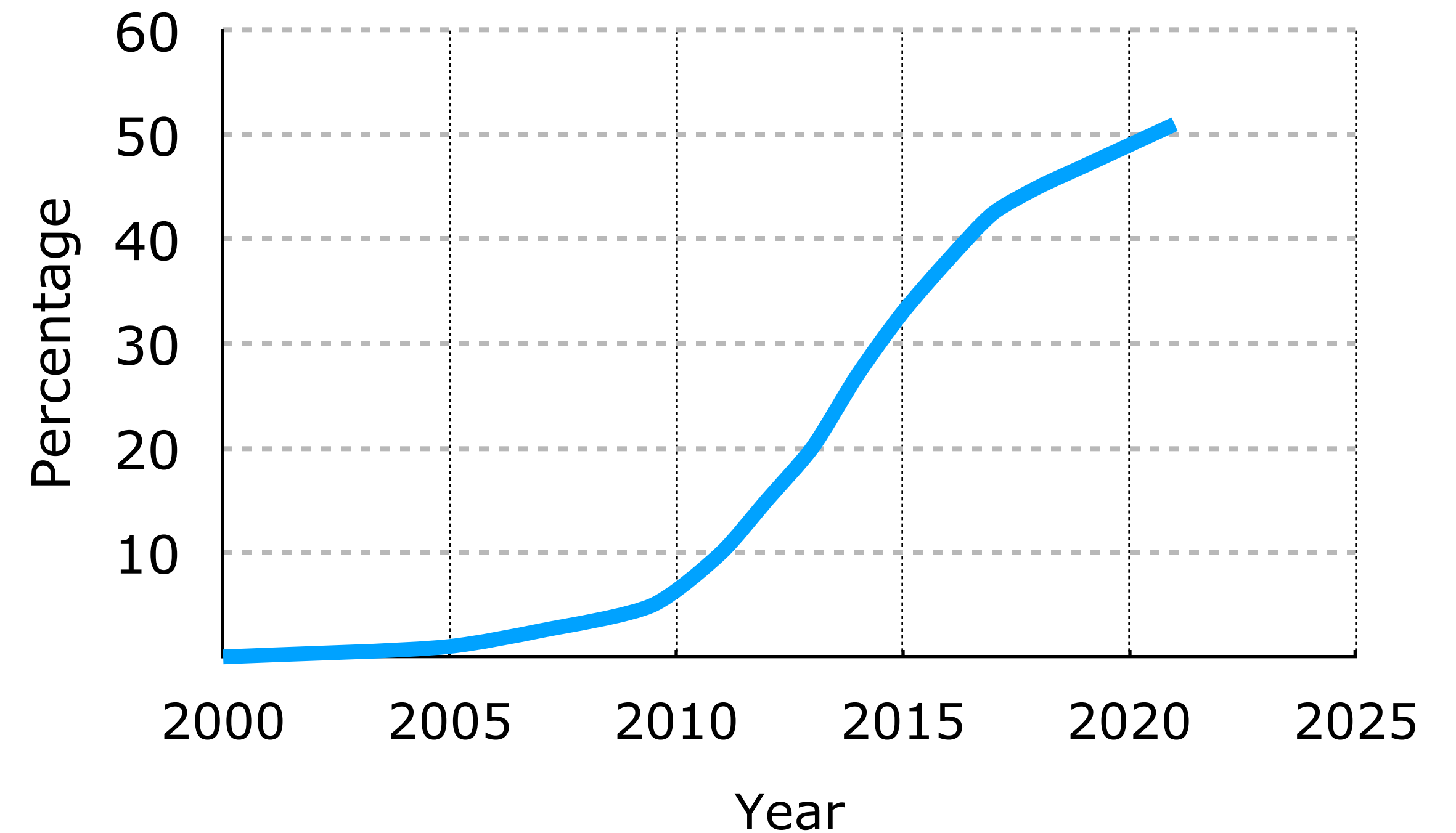




# Federated Learning



Percentage of world population with a smartphone (Data: Business Wire)



Communication cost > computation cost!

# Federated Learning: Collaborative Machine Learning without Centralized Training Data

April 6, 2017 · Posted by Brendan McMahan and Daniel Ramage, Research Scientists

Engineering at Meta

POSTED ON JUNE 14, 2022 TO [AI RESEARCH](#), [ML APPLICATIONS](#), [PRODUCTION ENGINEERING](#), [SECURITY](#)

## Applying federated learning to protect data on mobile devices

Federated Learning for Postoperative Segmentation of  
Treated glioblastoma (FL-PoS)

Federated learning in healthcare: the  
future of collaborative clinical and  
biomedical research



# How Apple personalizes Siri without hoovering up your data

The tech giant is using privacy-preserving machine learning to improve its voice assistant while keeping your data on your phone.

By Karen Hao

December 11, 2019



## IBM Federated Learning



**BANKING & PAYMENTS**

## Tencent's WeBank applying “federated learning” in A.I.

China's first mobile bank, Tencent's WeBank, is partnering with a H.K. startup to access decentralized sources of data.

Published 5 years ago on July 29, 2019



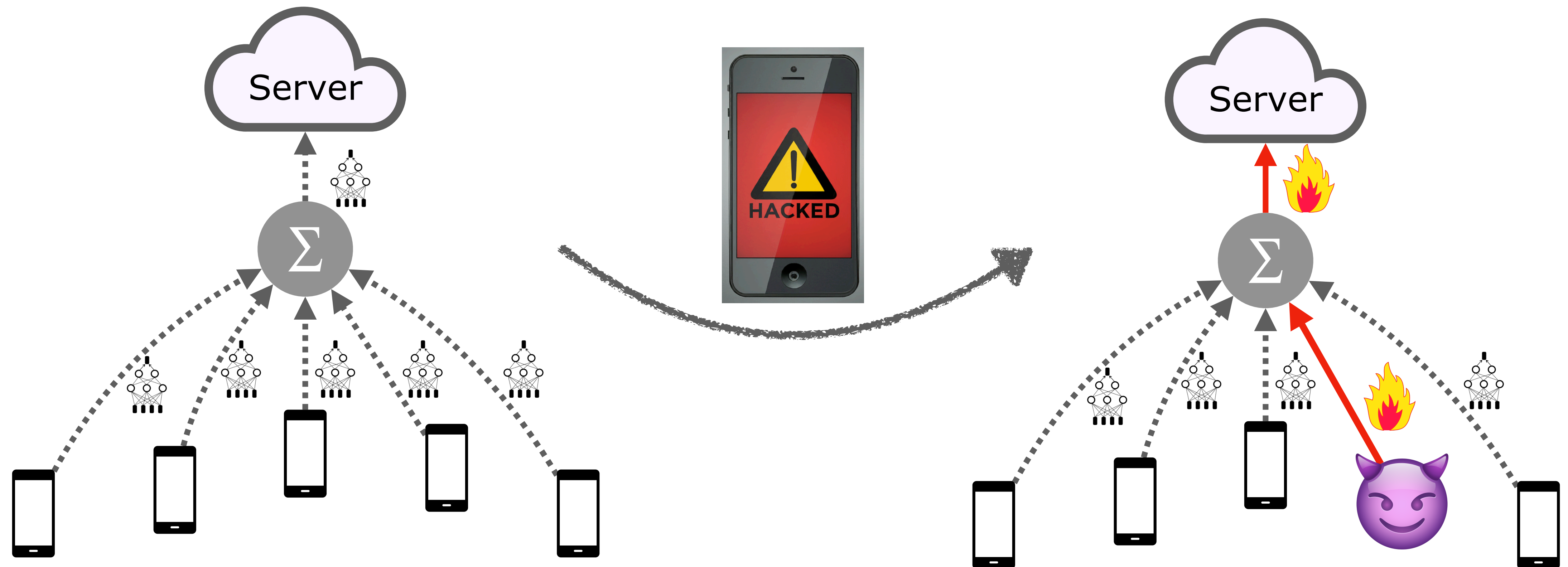
[PRODUCTS](#) [SUPPORT](#) [SOLUTIONS](#) [DEVELOPERS](#) [PARTNERS](#) [FOUNDRY](#)

[Developers](#) / [Topics & Technologies](#) ▾ / [Open Ecosystem](#) ▾ / [Try Federated Learning with OpenFL](#)

Try Federated Learning with OpenFL



**Challenge:**  
Training is *not robust* to potentially *malicious* clients

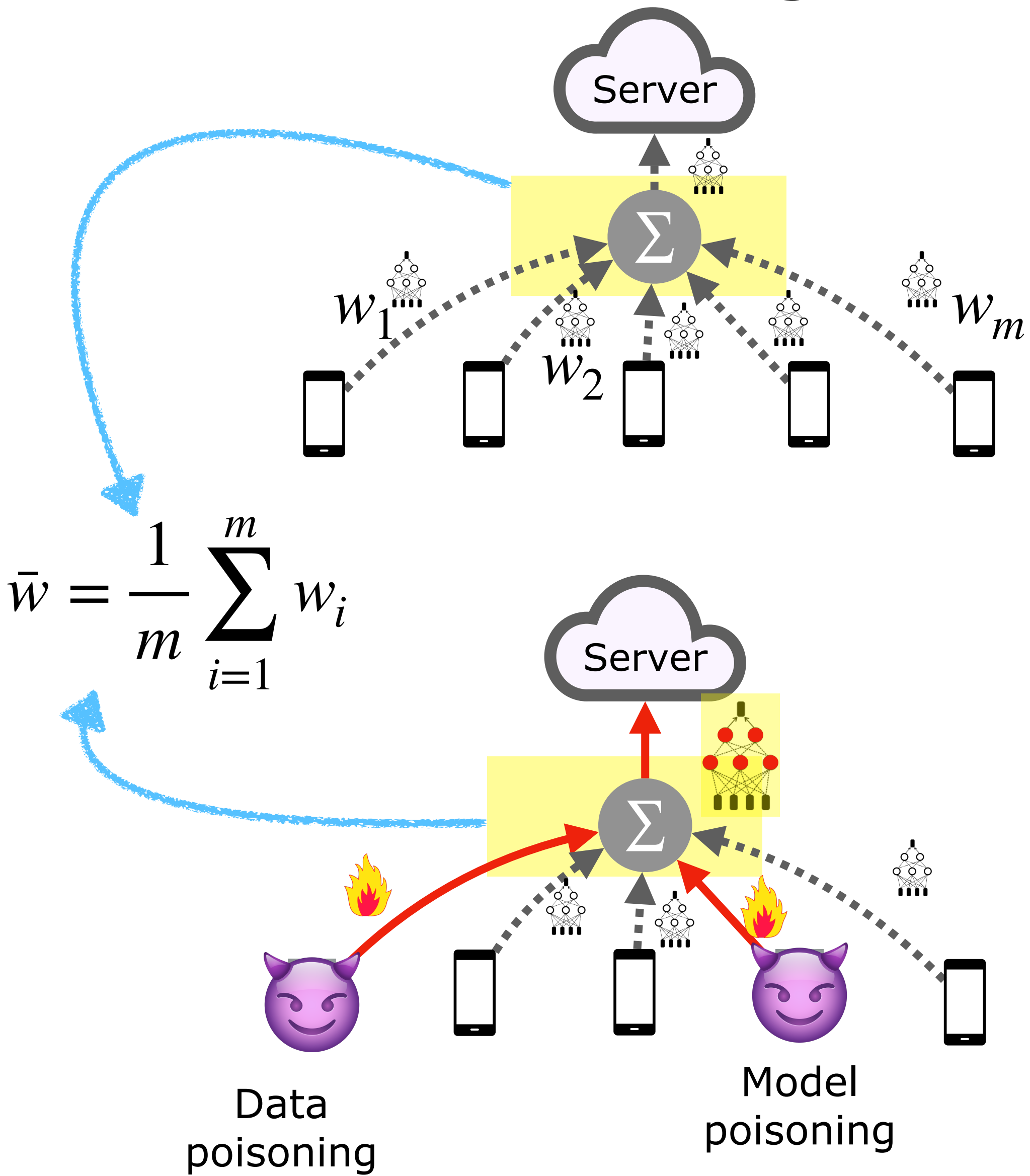


# Alexa and Siri Can Hear This Hidden Command. You Can't.

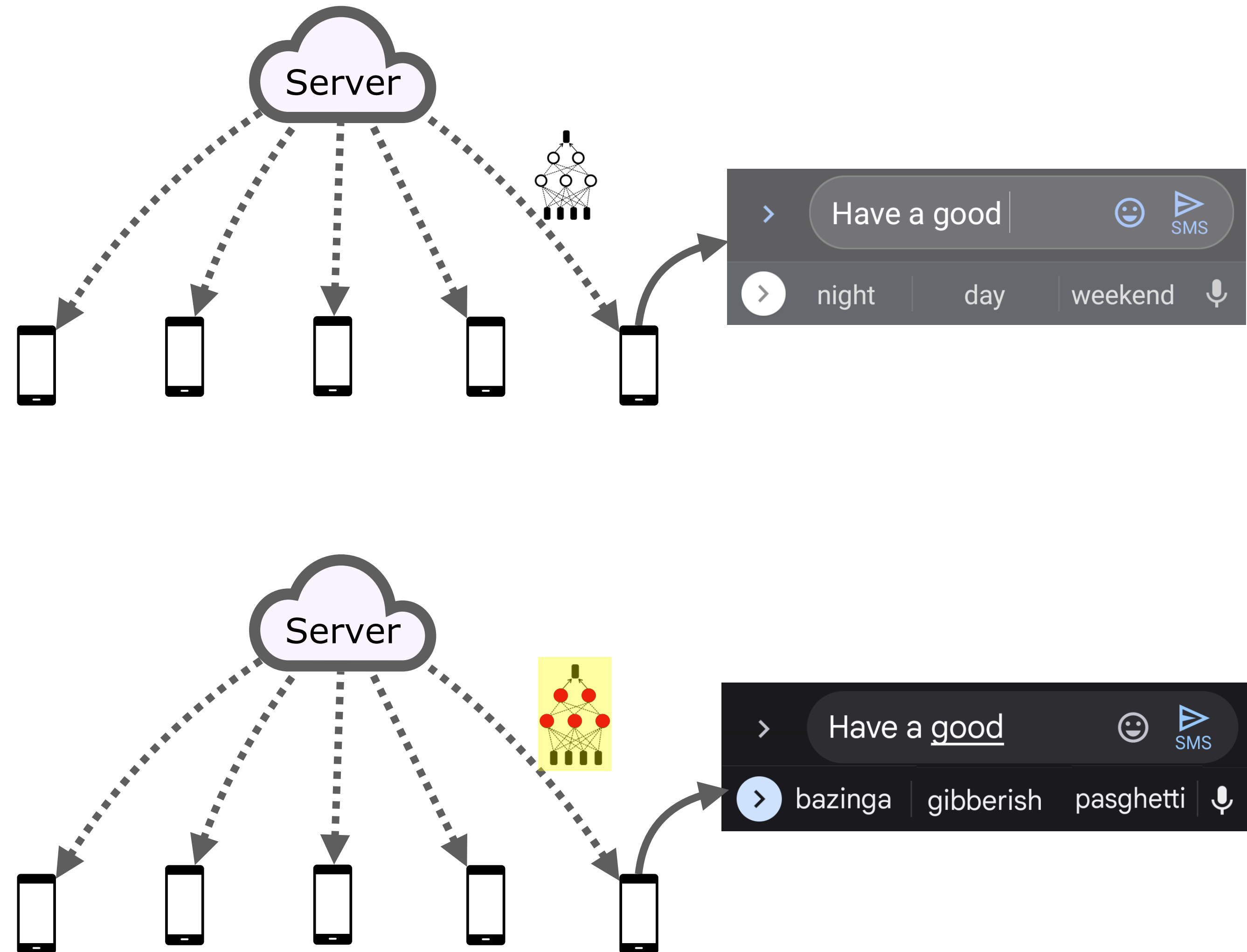
Researchers can now send secret audio instructions  
undetectable to the human ear to Apple's Siri, Amazon's  
Alexa and Google's Assistant.



# Training



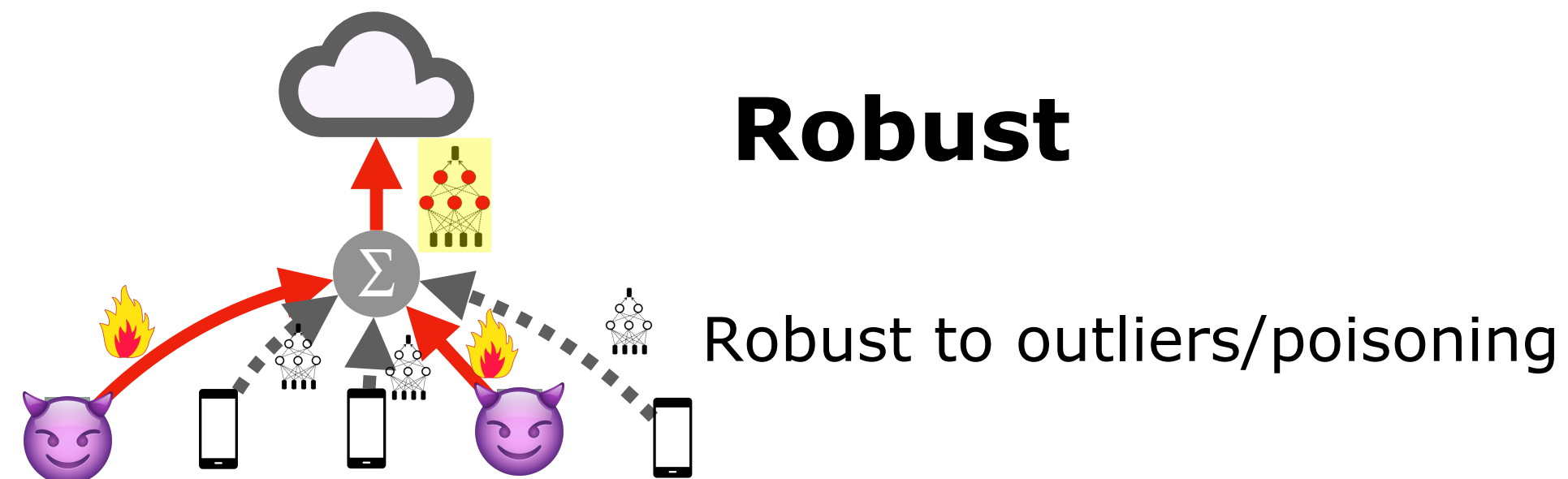
# Deployment



Usual mean aggregation is ***not robust*** to corruptions  $\Rightarrow$  **Poor predictions!**

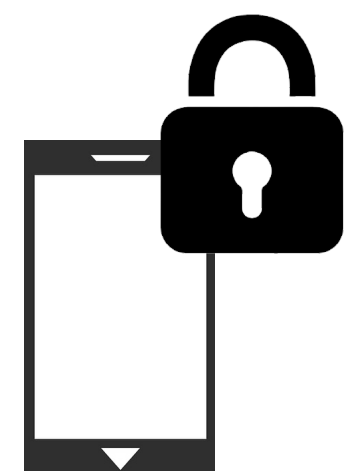
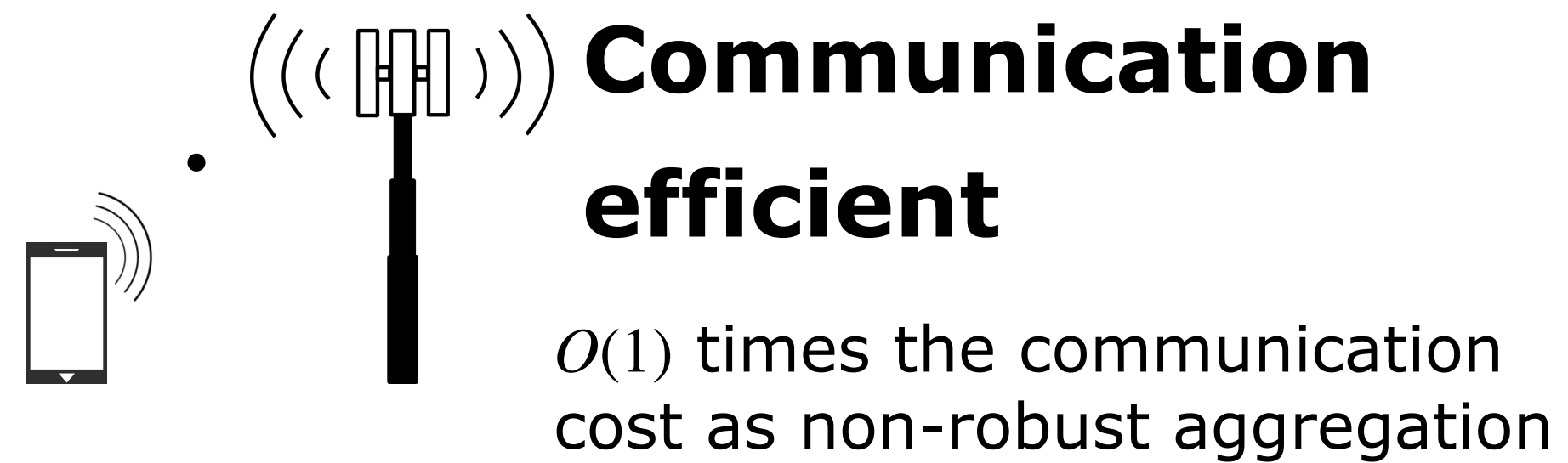
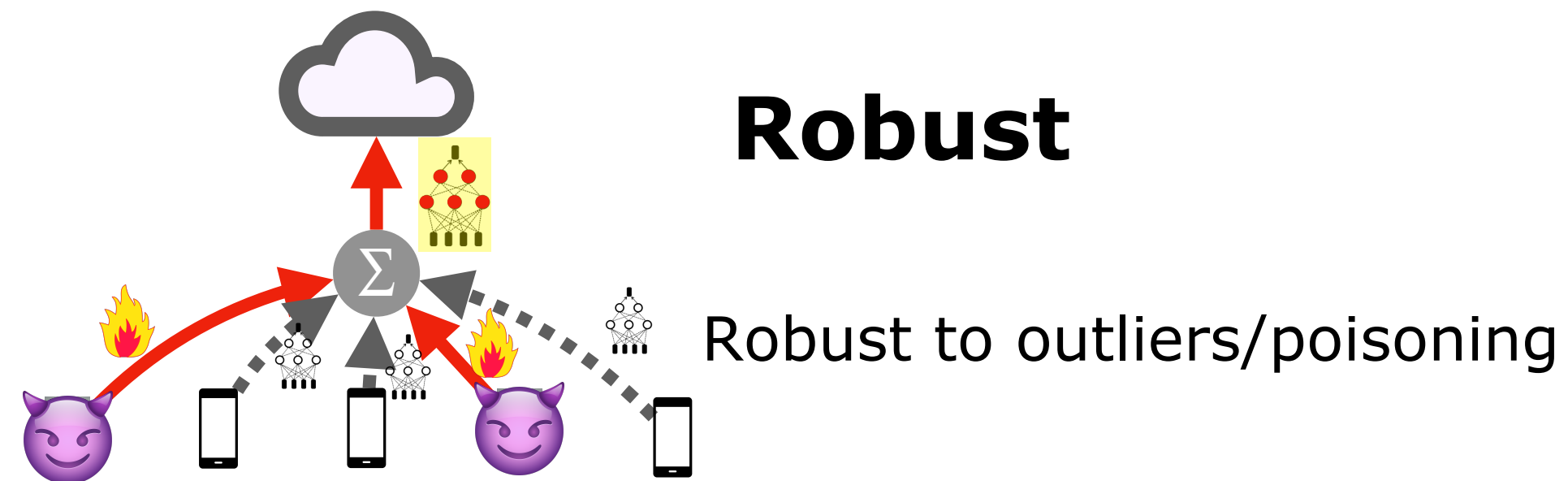
# ***Usual approach*** (Direct)

# ***Our approach*** (Variational)



# *Usual approach* (Direct)

# *Our approach* (Variational)



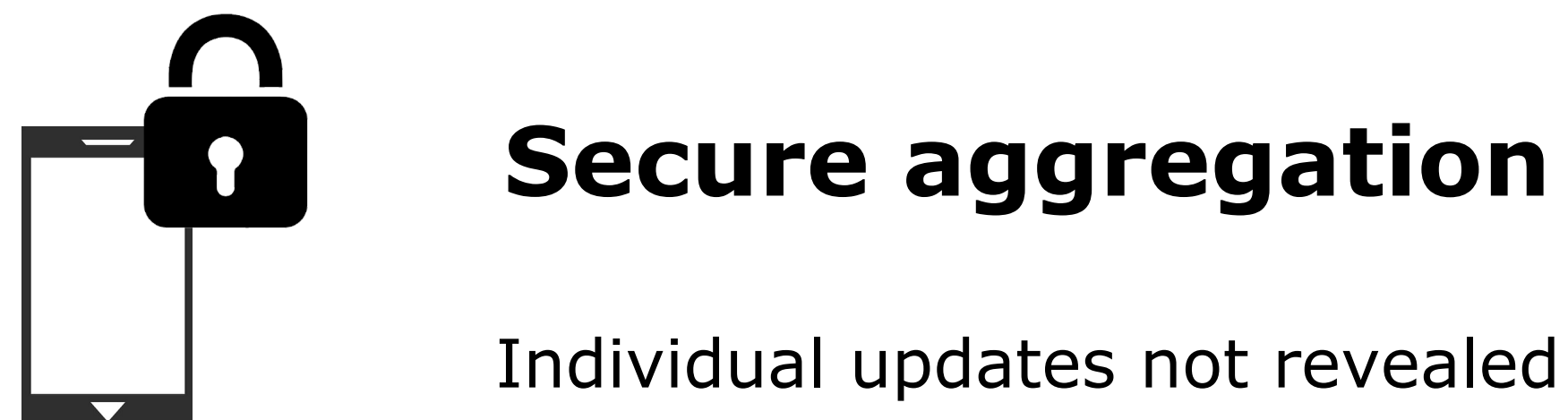
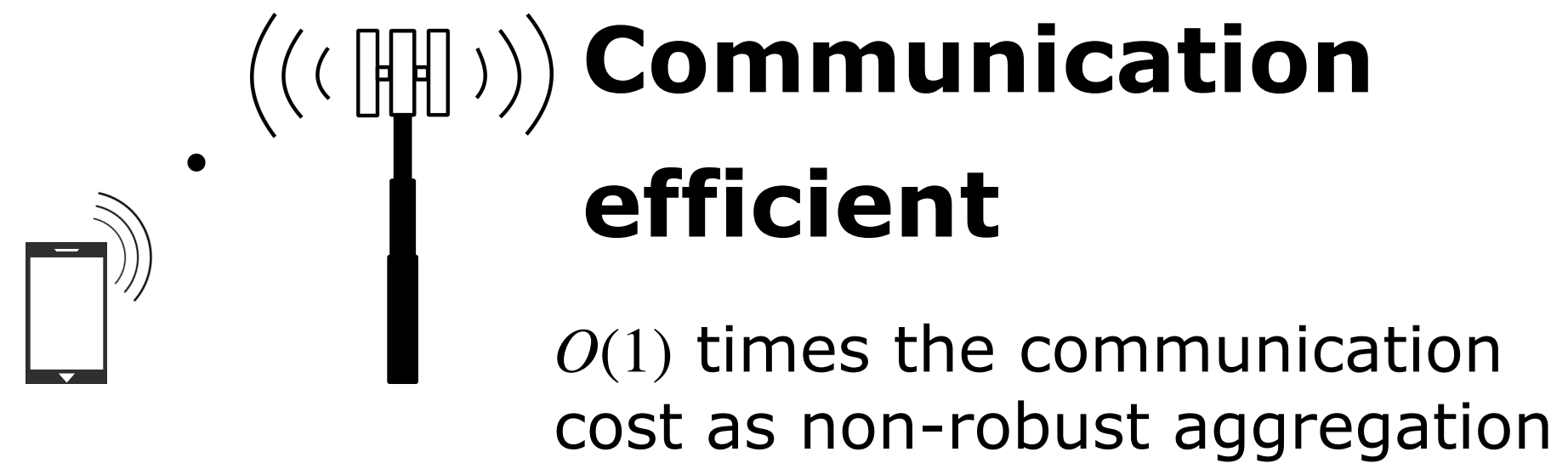
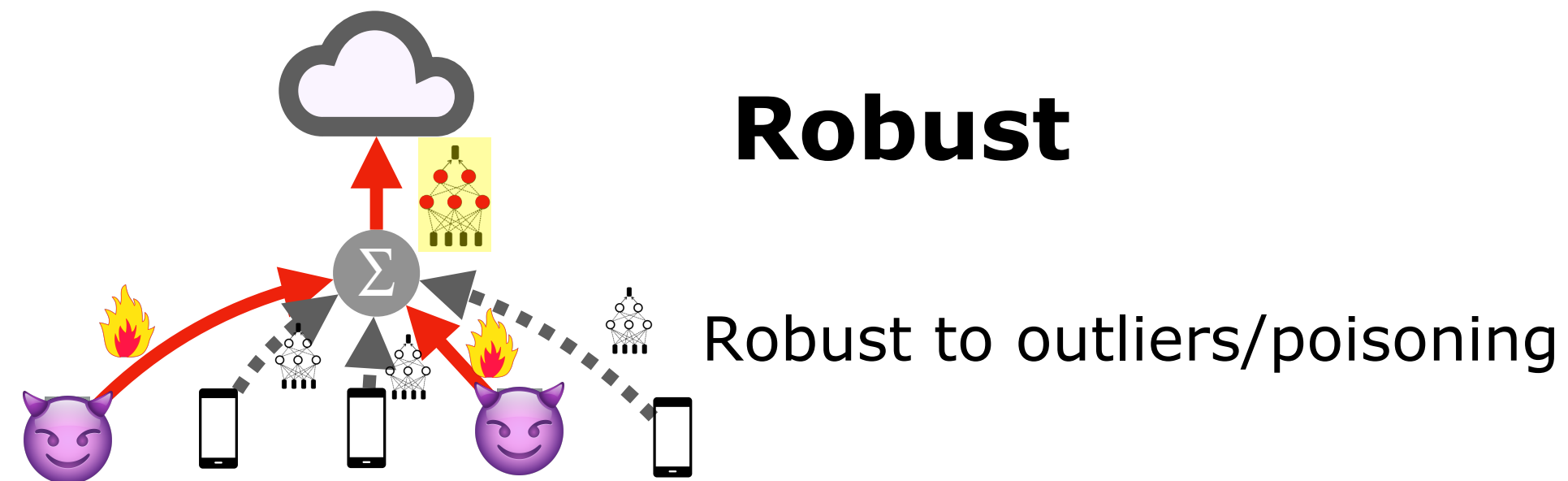
## **Secure aggregation**

Individual updates not revealed



# *Usual approach* (Direct)

# *Our approach* (Variational)

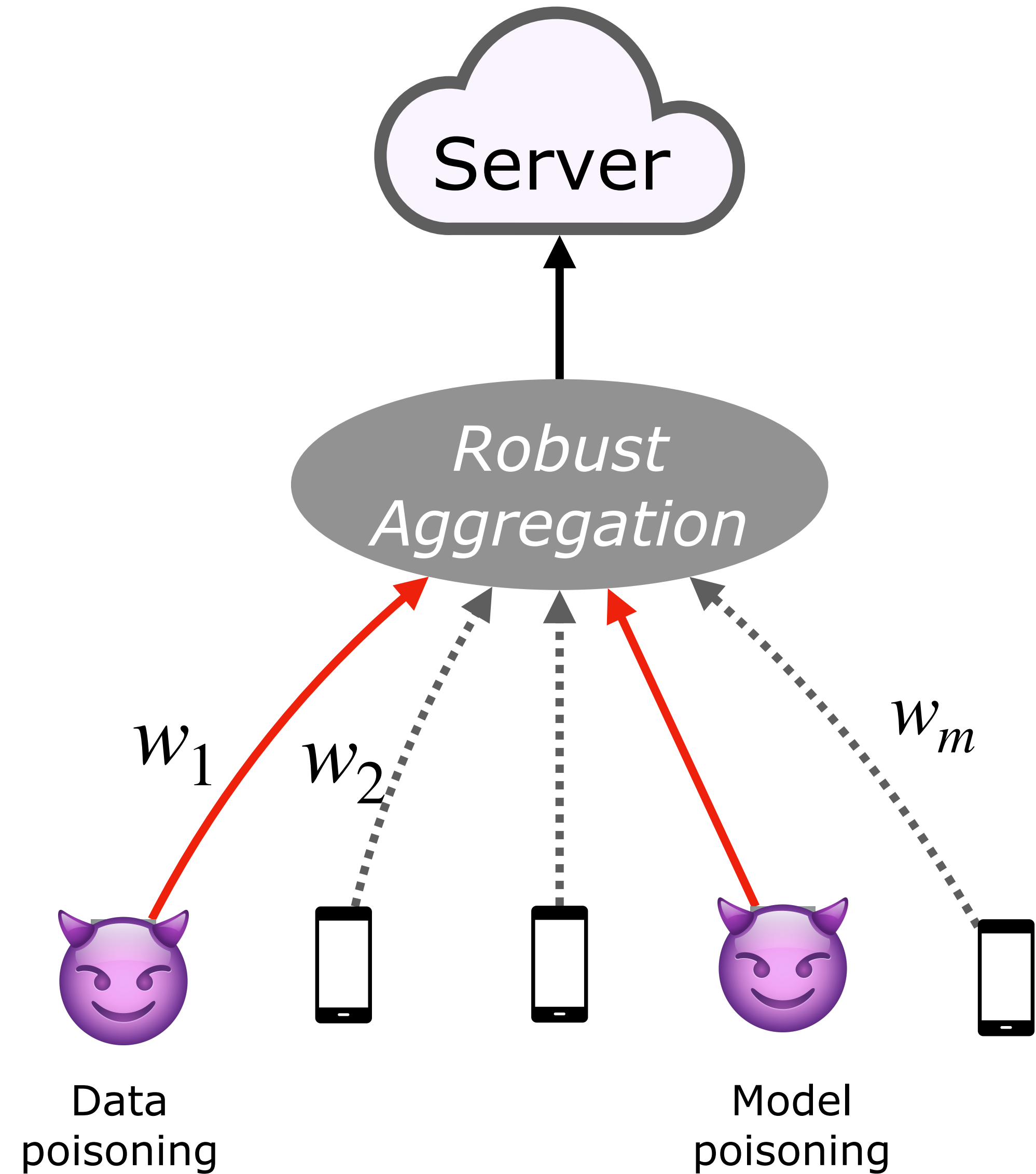


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# Robust aggregation approach

$w_1, \dots, w_m$ : updates sent by the clients







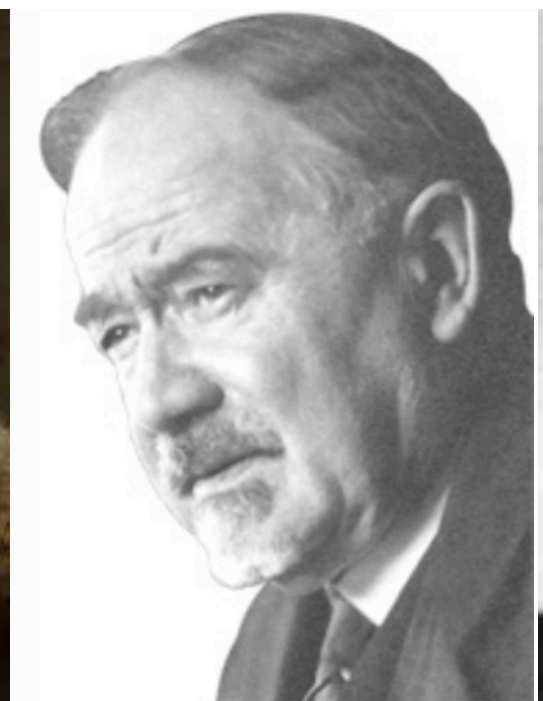
## Facility location



Fermat  
(~1600s)



Torricelli

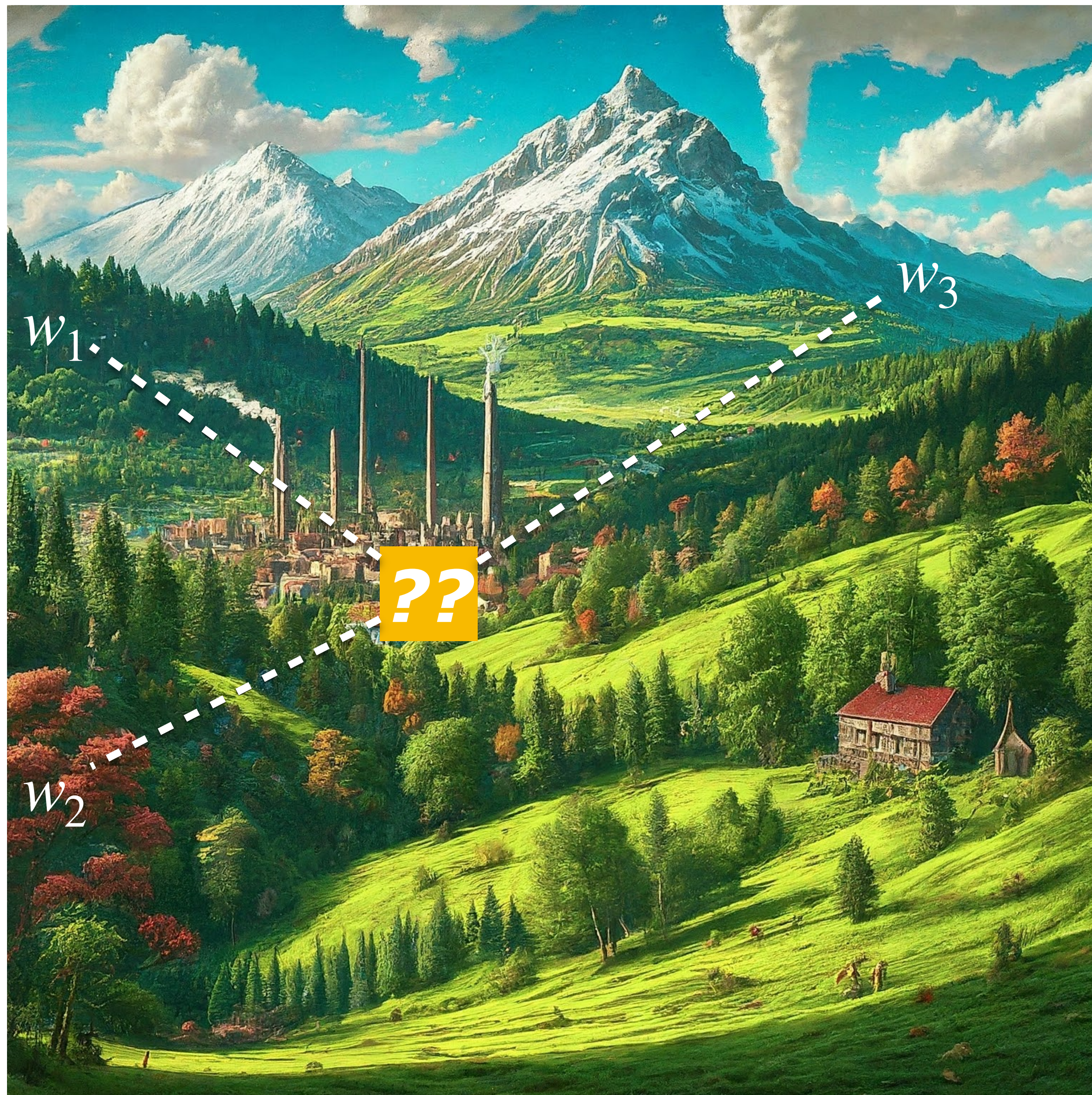


Weber  
(1909)



Fréchet  
(~1940s)





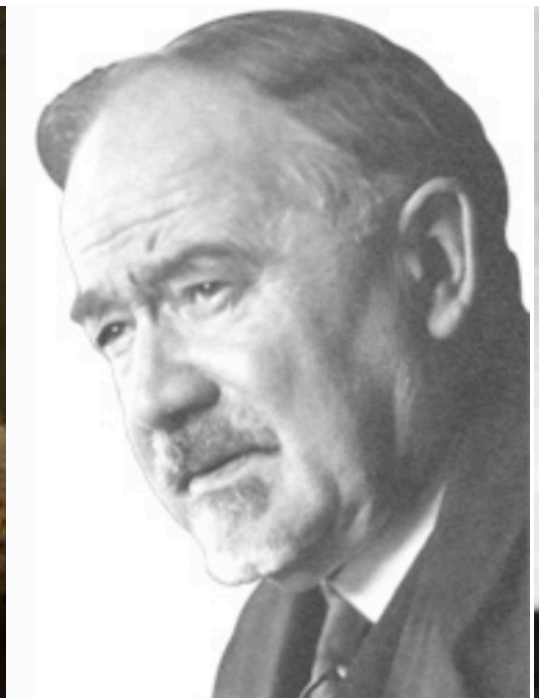
## Facility location



Fermat  
(~1600s)



Torricelli



Weber  
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Fréchet  
(~1940s)





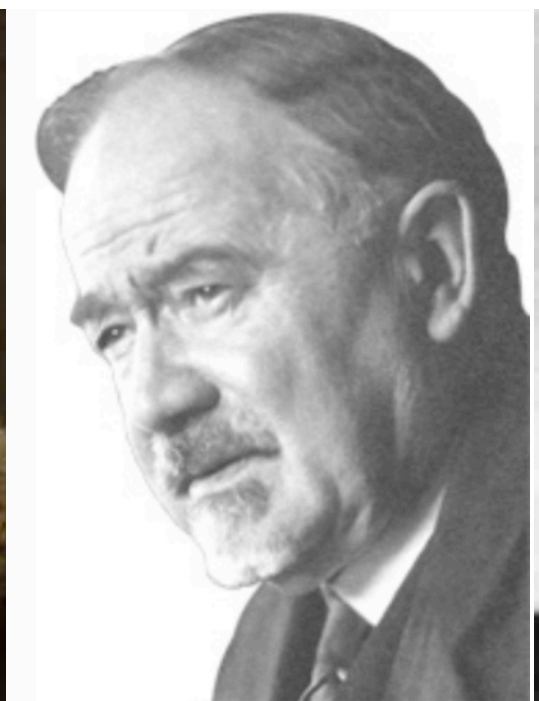
## Facility location



Fermat  
(~1600s)



Torricelli

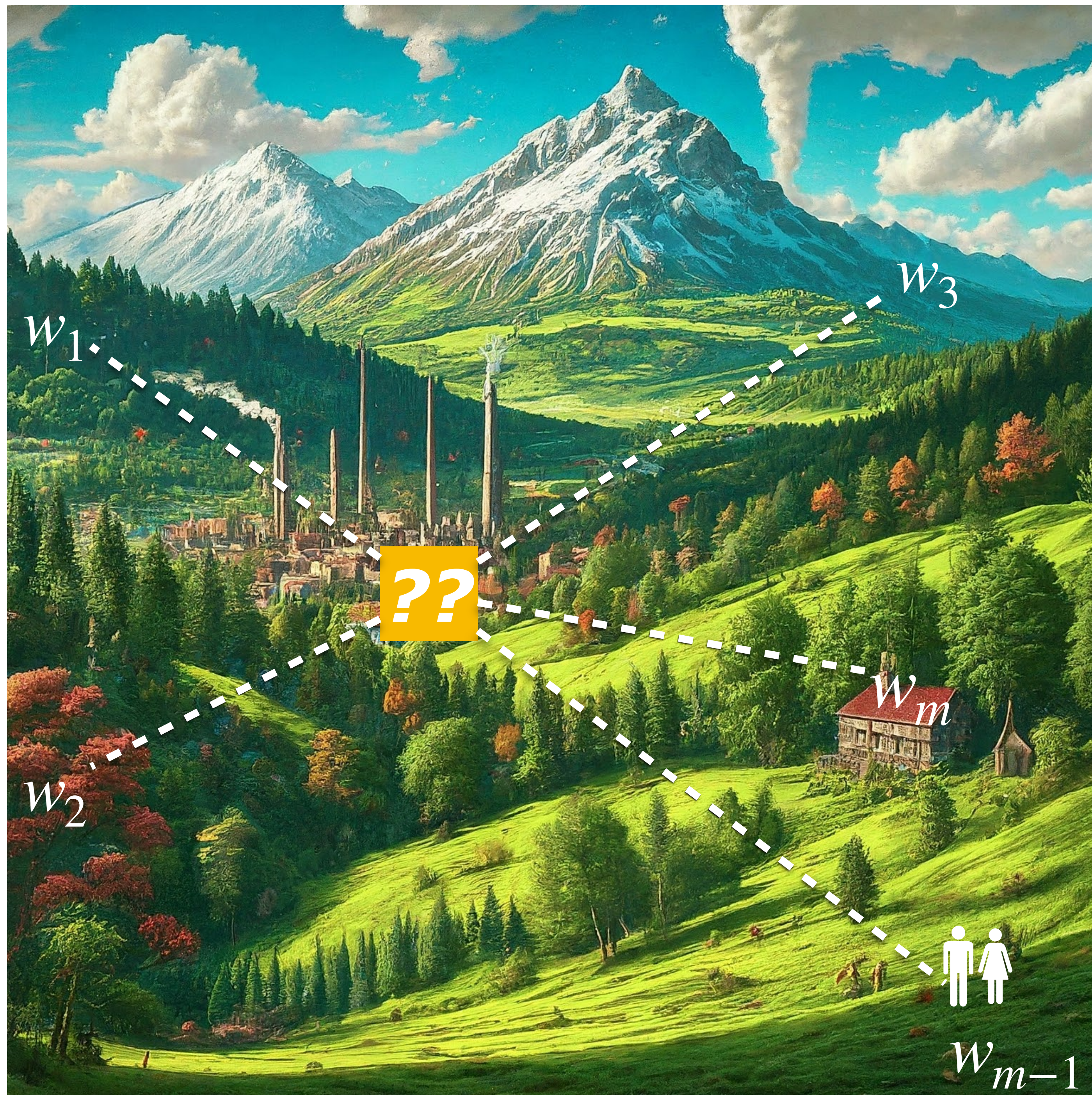


Weber  
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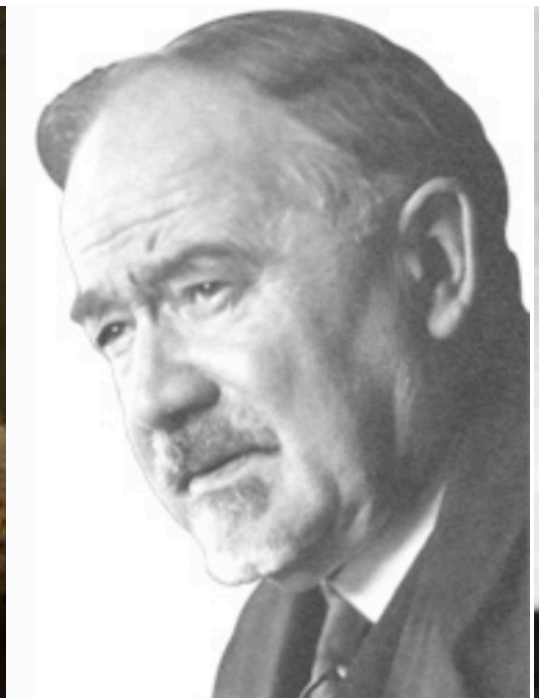
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Torricelli

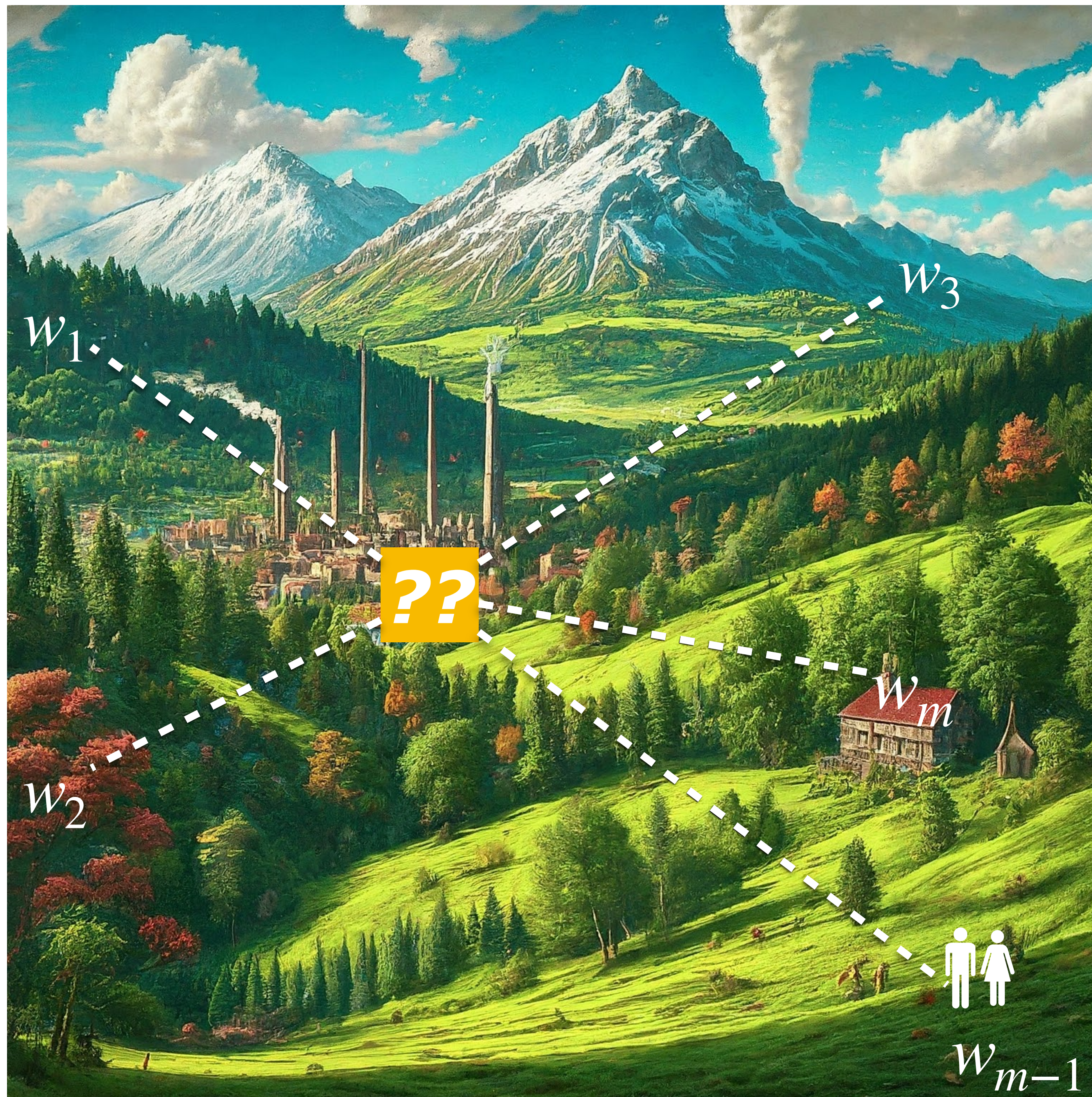


Weber  
(1909)



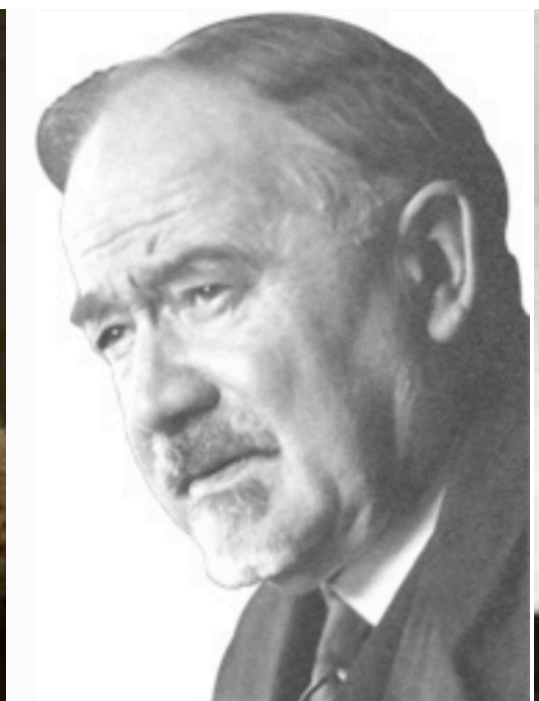
Fréchet  
(~1940s)





# Geometric Median / Spatial Median / $L_1$ Median / Facility location

$$\text{GM}(w_1, \dots, w_m) = \arg \min_z \left\{ \sum_{i=1}^m \|z - w_i\|_2 \right\}$$



Fermat

(~1600s)

Torricelli

Weber

(1909)

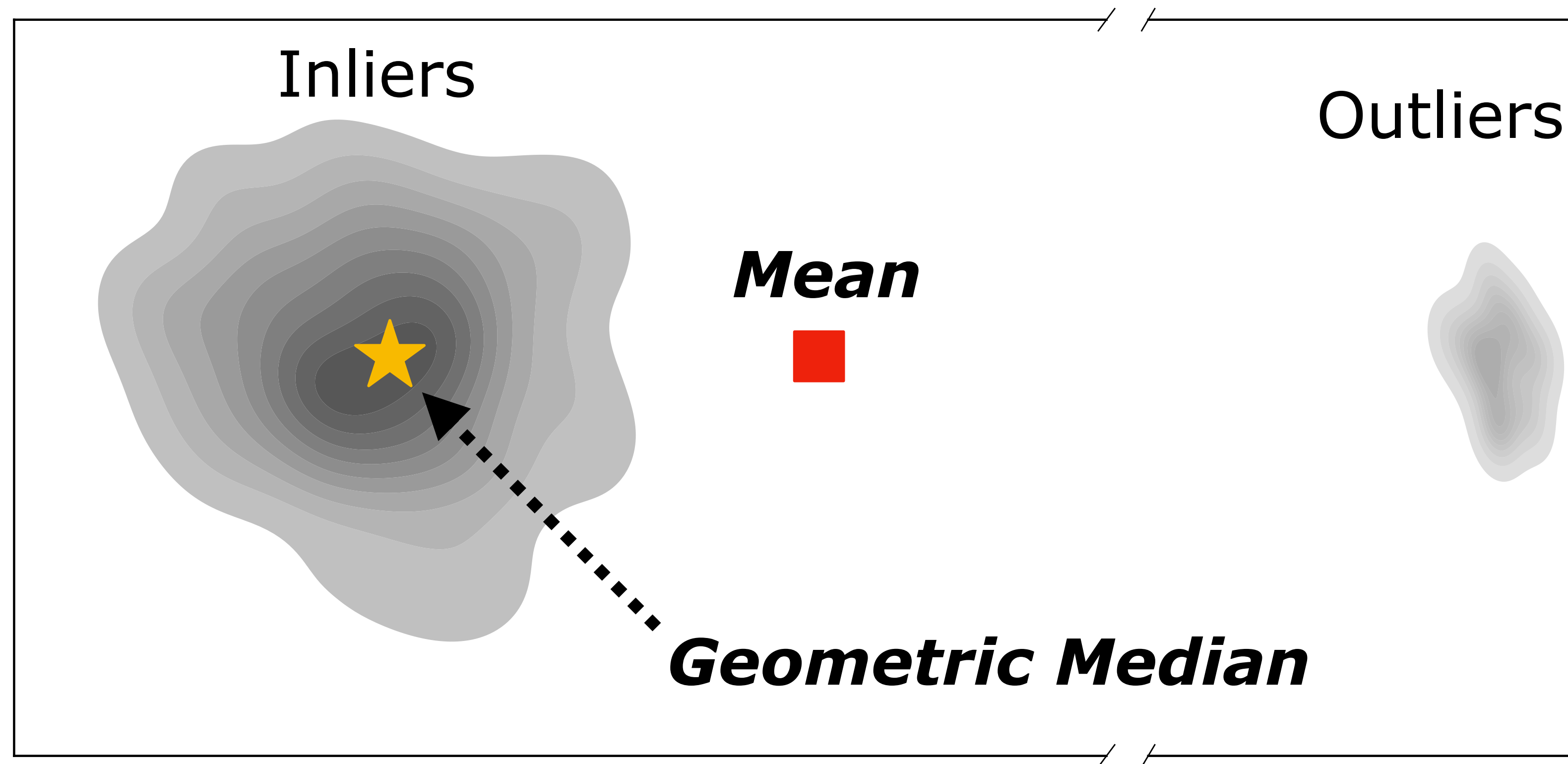
Fréchet

(~1940s)



# Robustness: Breakdown point = $1/2$

(In **1D**, we have that *geometric median*  $\equiv$  *usual median*)



Nemirovski & Yudin (1983) | Jerrum, Valiant & Vazirani (1986) | Lopuhaa & Rousseeuw (1991)  
Hsu & Sabata (2013) | Minsker (2015) | Lugosi, Gabor & Mendelson (2019) | Lecué & Lerasle (2020)

# Smoothed Weiszfeld Algorithm

Weiszfeld (1937). **Sur le point par lequel la somme des distances de  $n$  points donnees est minimum.** *Tohoku Mathematical Journal*.

Compute new weights  $\beta_{i,t} = \frac{1}{\max\{\|z_t - w_i\|_2, \nu\}}$  & Reweighted average  $z_{t+1} = \frac{\sum_i \beta_{i,t} w_i}{\sum_i \beta_{i,t}}$



Weiszfeld a.k.a. Vázsonyi (1916-2003)



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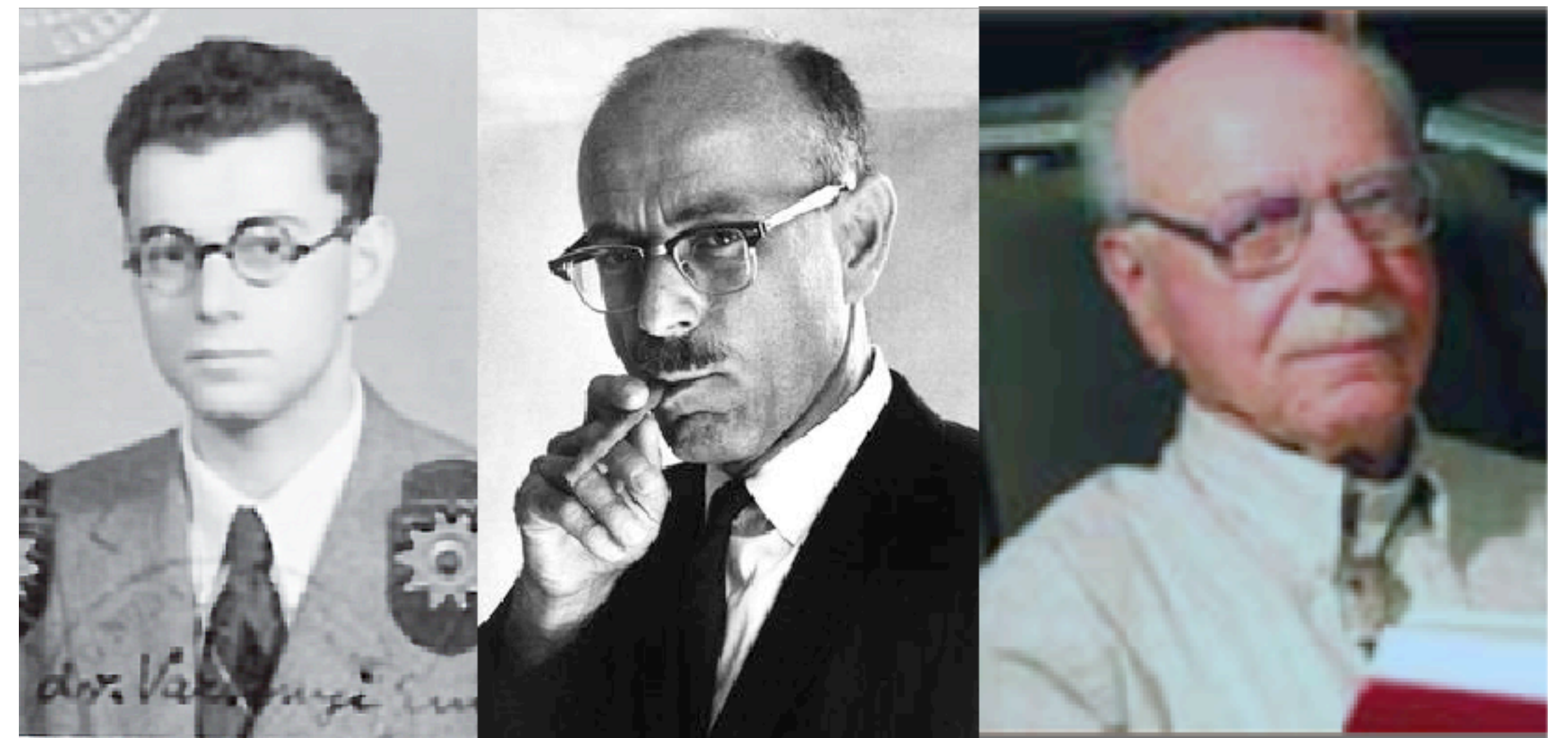
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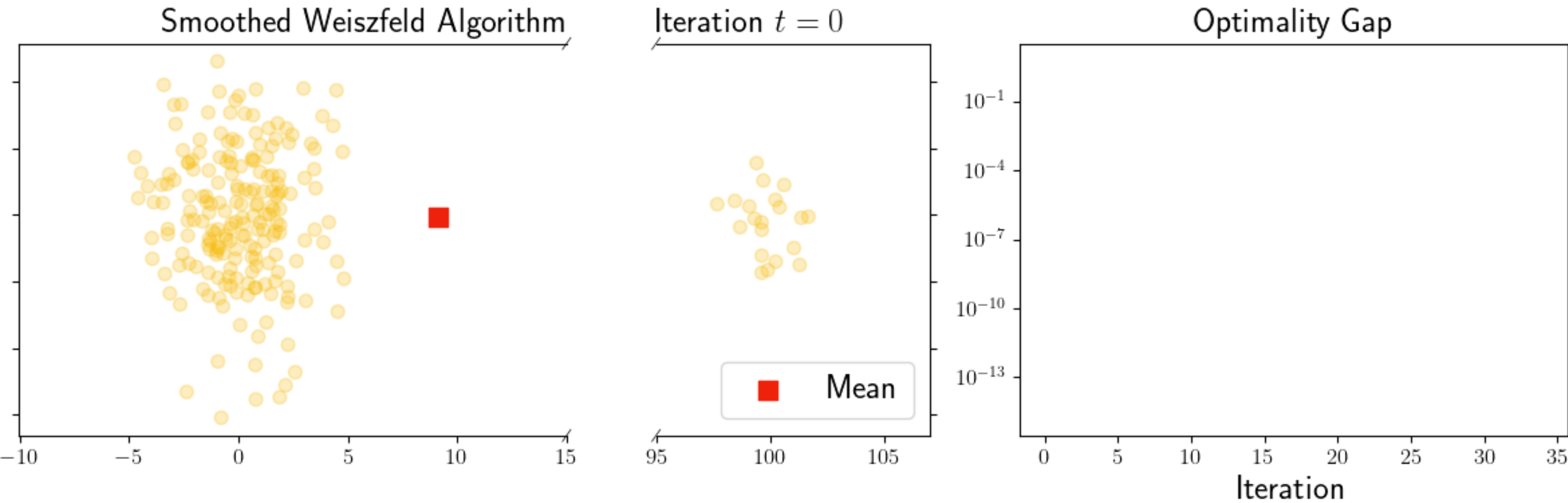


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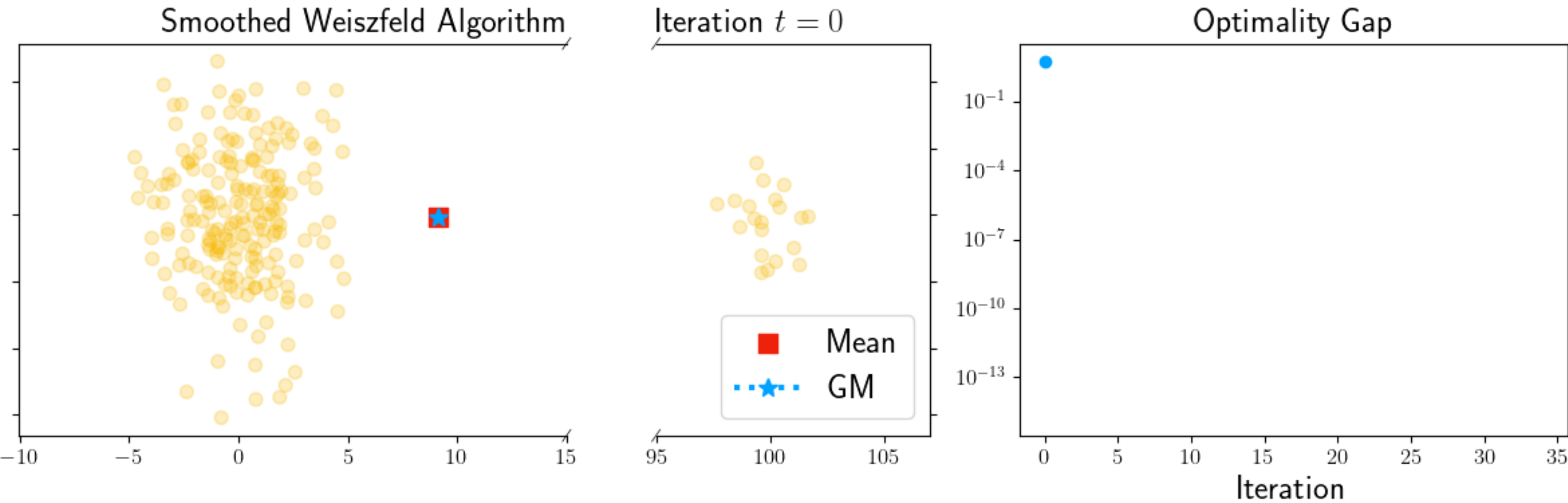




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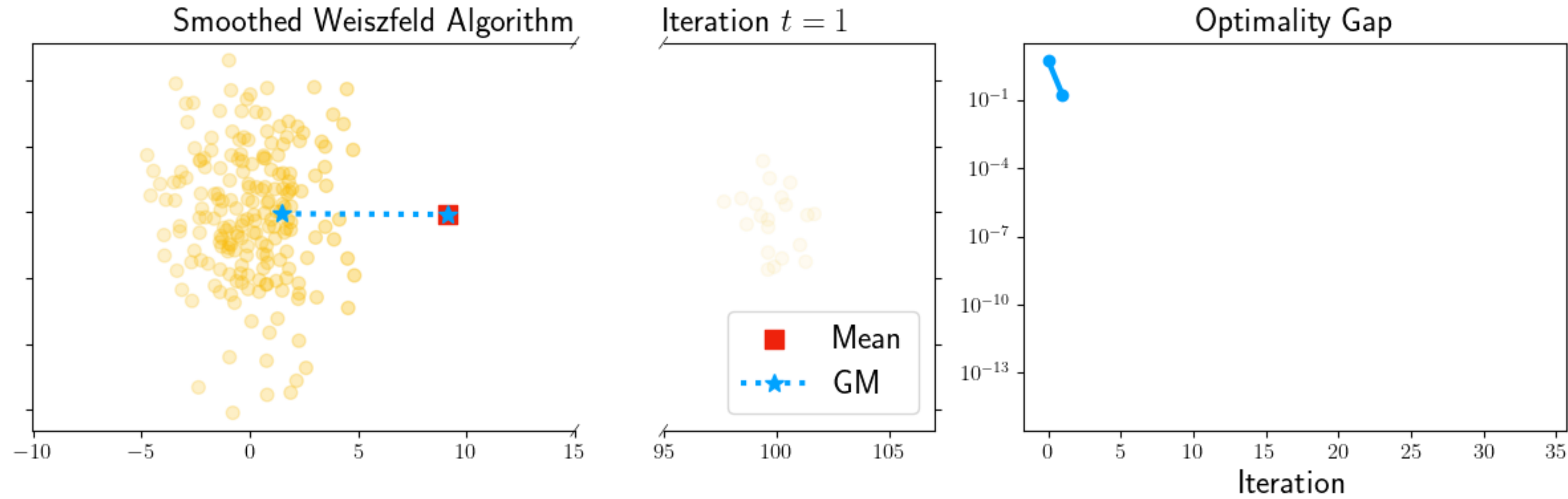




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Weiszfeld (1937). **Sur le point par lequel la somme des distances de  $n$  points donnees est minimum.** *Tohoku Mathematical Journal*.

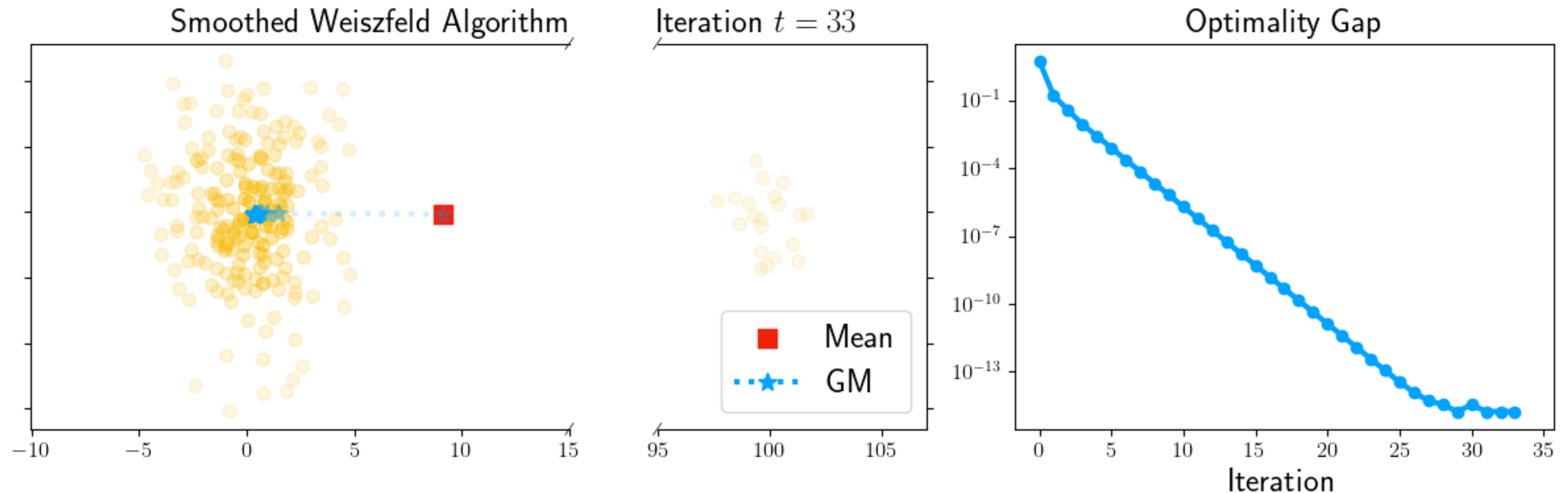
Compute new weights  $\beta_{i,t} = \frac{1}{\max\{\|z_t - w_i\|_2, \nu\}}$       &      Reweighted average  $z_{t+1} = \frac{\sum_i \beta_{i,t} w_i}{\sum_i \beta_{i,t}}$



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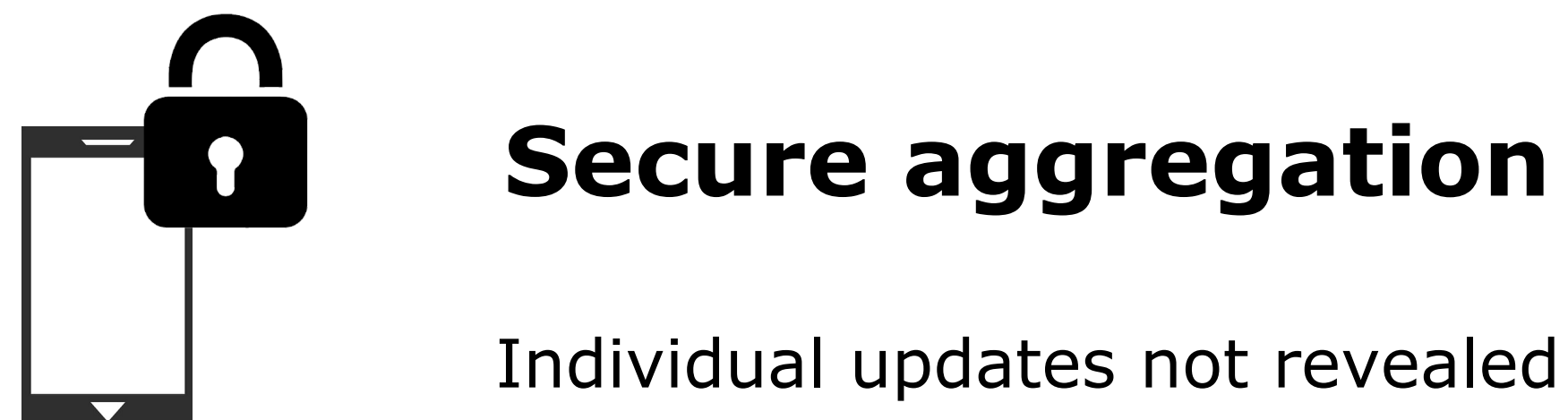
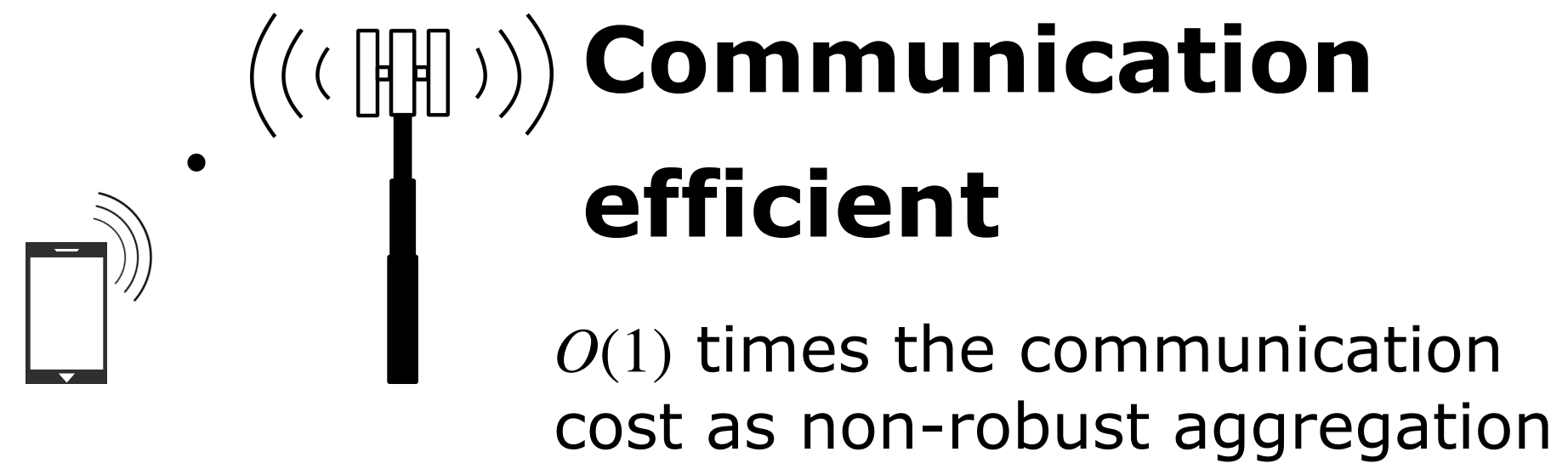
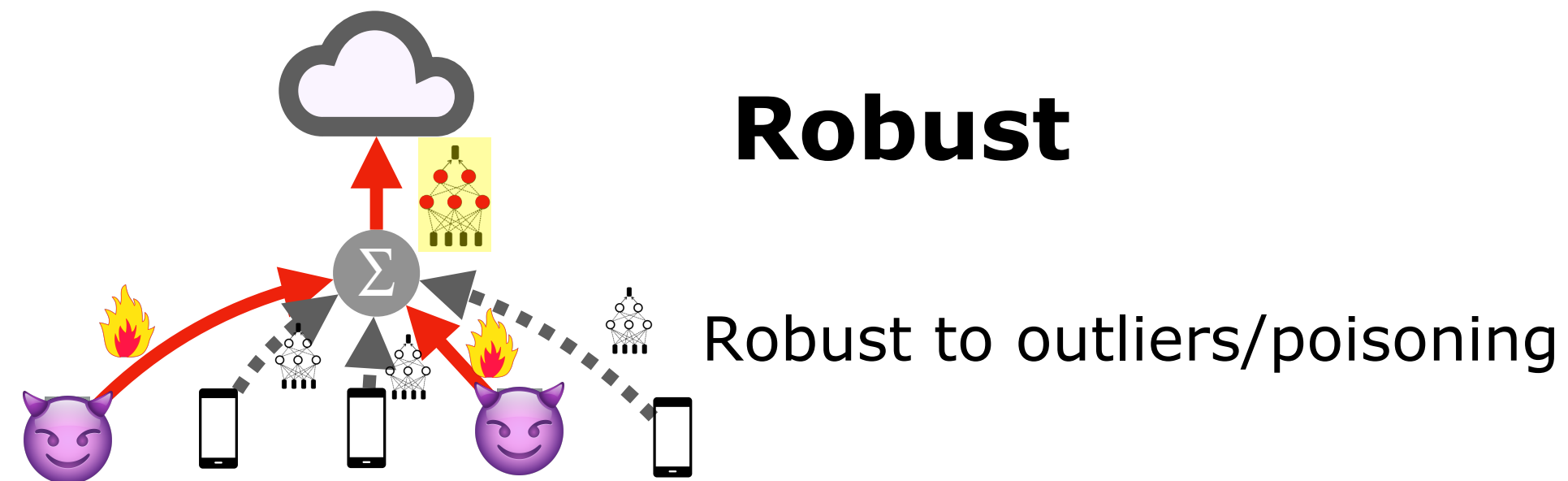
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# *Usual approach* (Direct)

# *Our approach* (Variational)

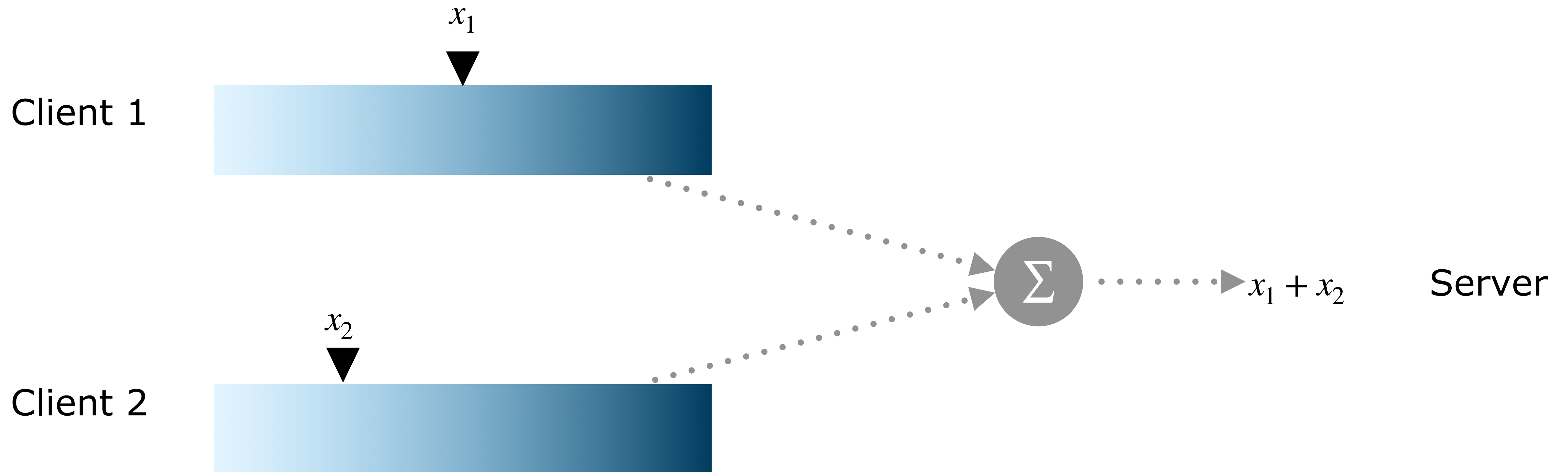


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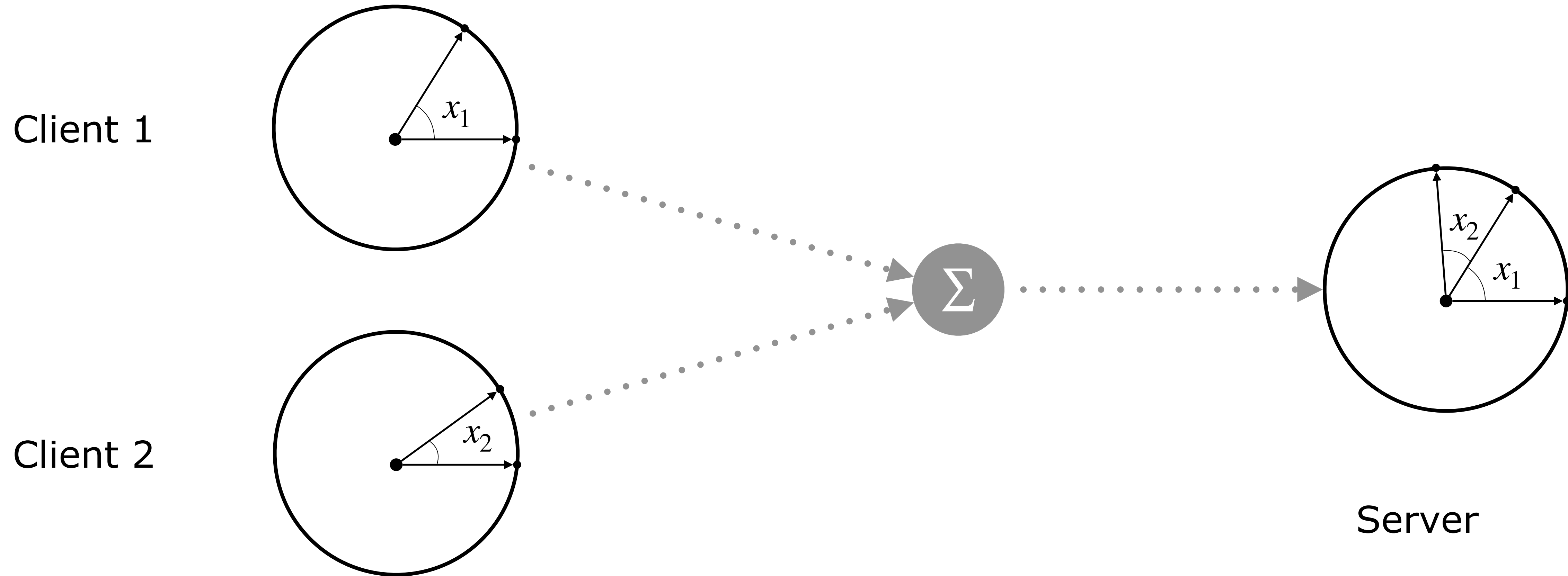
# Communication primitive: secure sum/average

Only reveal  $x_1 + x_2$  to the server without revealing  $x_1$  or  $x_2$



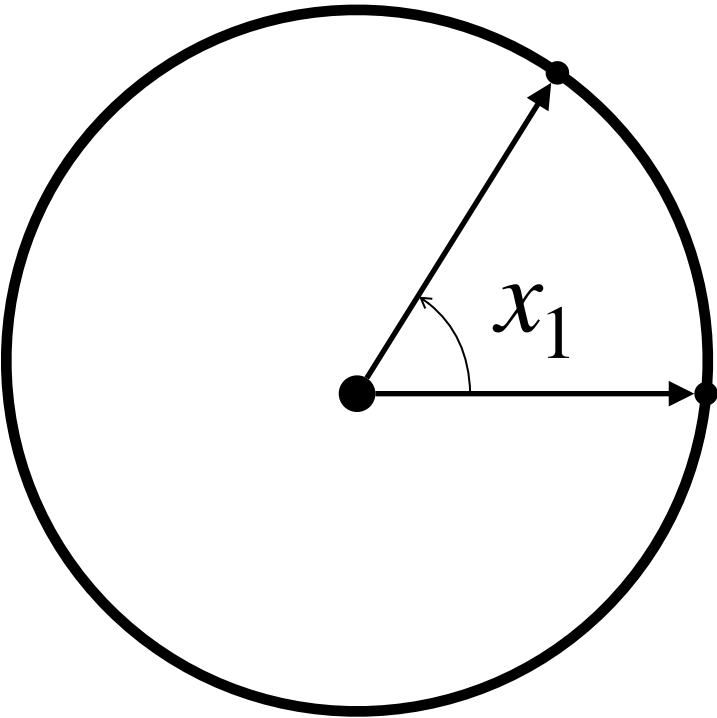


Perform all operations modulo  $M$

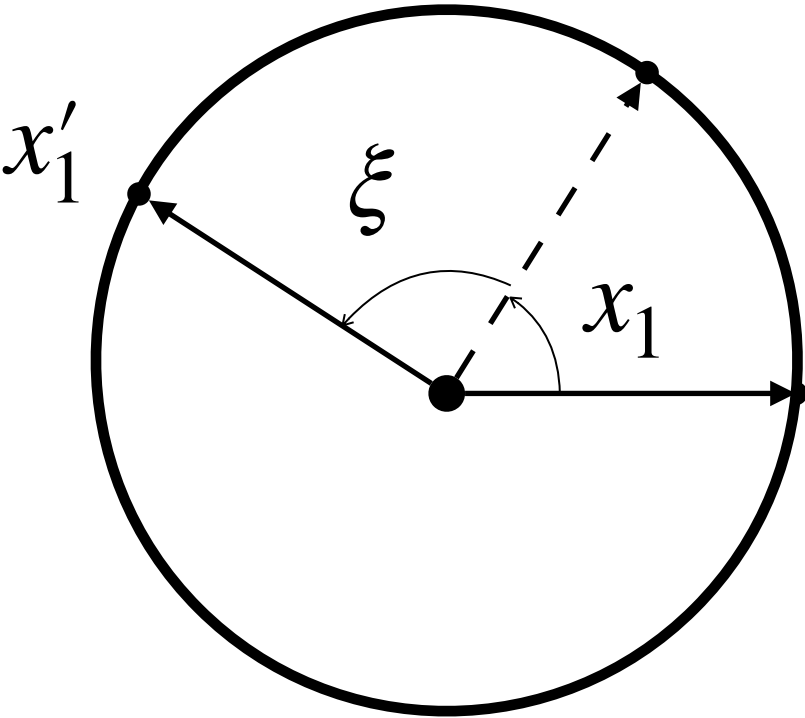


[Bonawitz et al. CCS (2017), Bell et al. CCS (2020)]

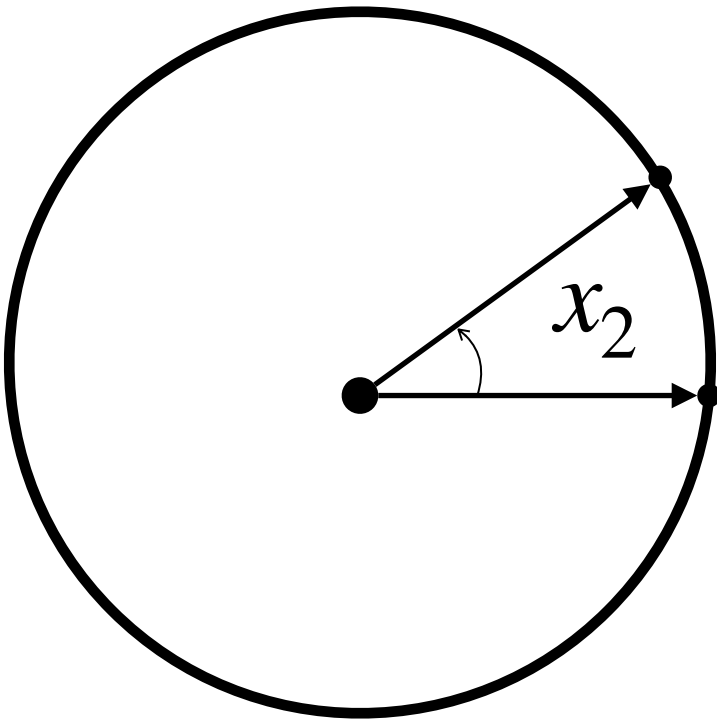
Client 1



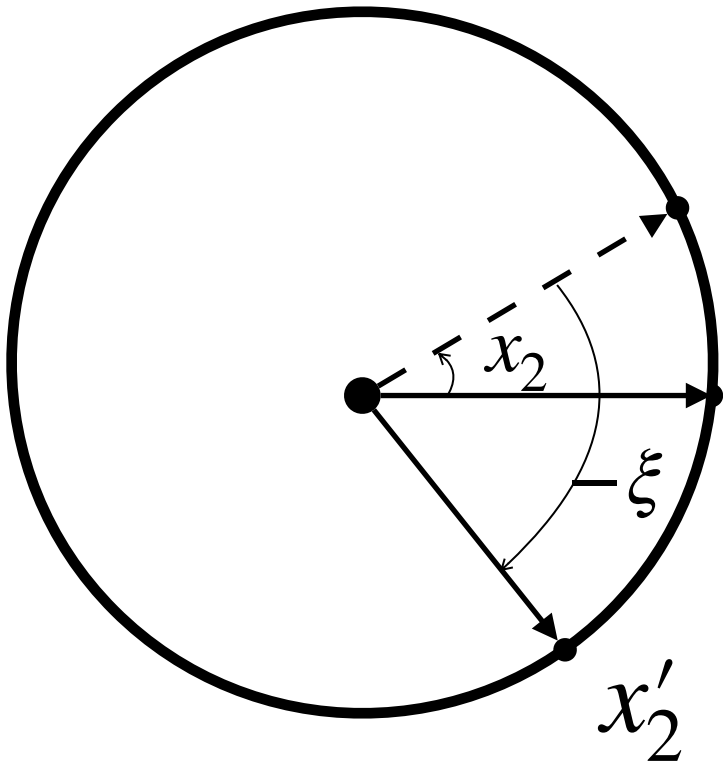
$x'_1 = x_1 + \xi$   
.....▶



Client 2



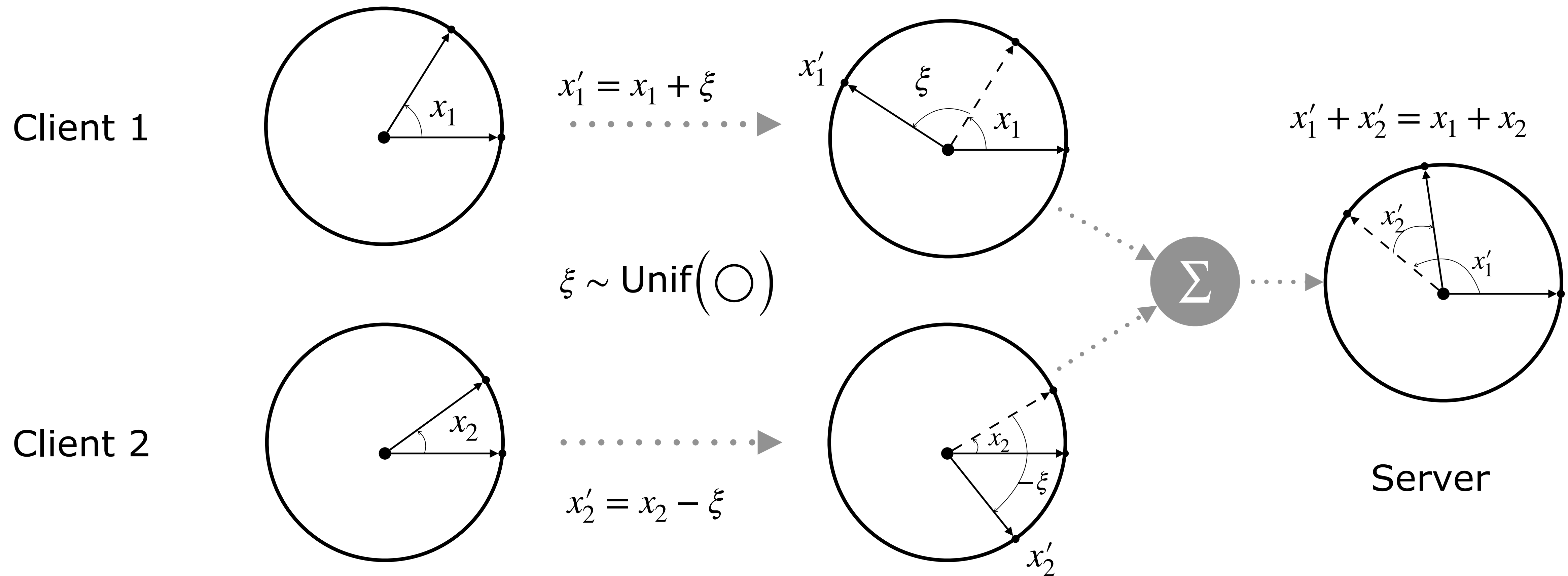
.....▶  
 $x'_2 = x_2 - \xi$



[Bonawitz et al. CCS (2017), Bell et al. CCS (2020)]

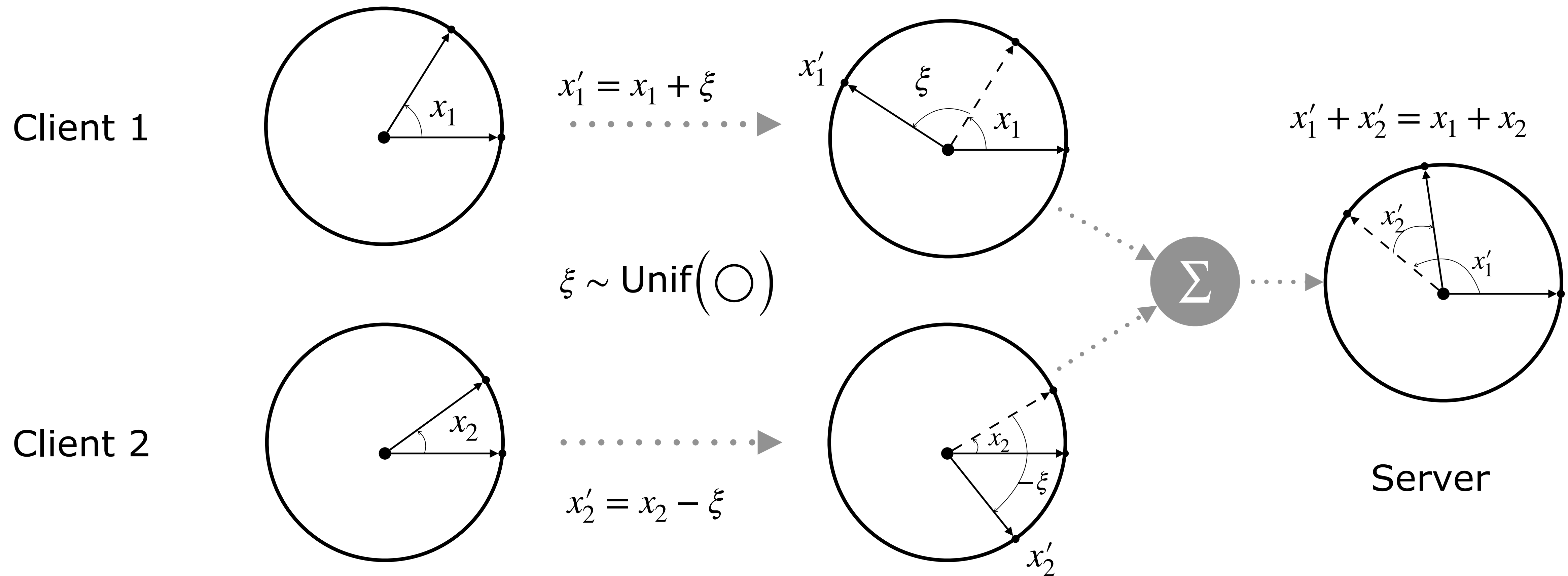


Server only sees  $x'_1, x'_2 \sim \text{Unif}(\bigcirc)$  but calculates the correct sum (and average)





Server only sees  $x'_1, x'_2 \sim \text{Unif}(\bigcirc)$  but calculates the correct sum (and average)



Total communication for  $m$  vectors in  $\mathbb{R}^d = O(m \log m + md)$  numbers



Server only sees  $x'_1, x'_2 \sim \text{Unif}(\bigcirc)$  but calculates the correct sum



Total communication for  $m$  vectors in  $\mathbb{R}^d = O(m \log m + md)$  numbers



# Smoothed Weiszfeld Algorithm

Weiszfeld (1937). **Sur le point par lequel la somme des distances de  $n$  points donnees est minimum.** *Tohoku Mathematical Journal*.

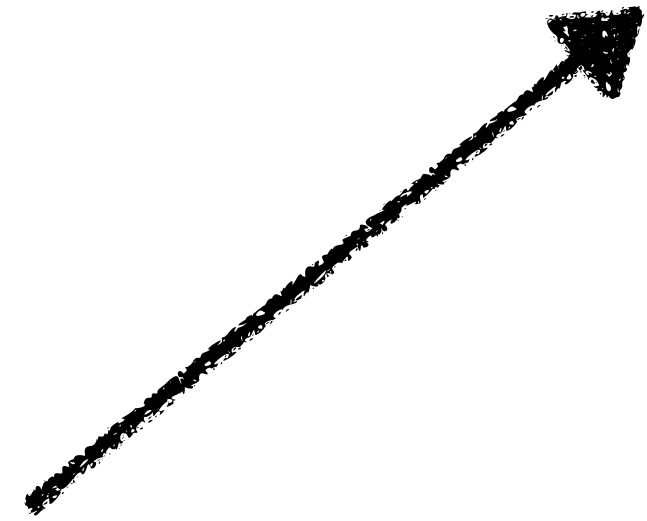
Compute new weights  $\beta_{i,t} = \frac{1}{\max\{\|z_t - w_i\|_2, \nu\}}$       &      Reweighted average  $z_{t+1} = \frac{\sum_i \beta_{i,t} w_i}{\sum_i \beta_{i,t}}$



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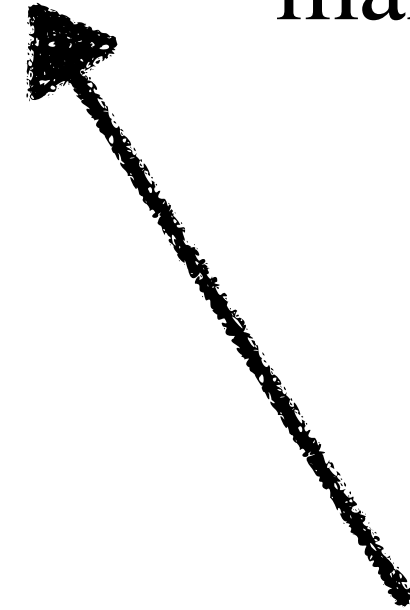
1. Server **broadcasts** current estimate  $z_t$  of the geometric median



# Smoothed Weiszfeld Algorithm

Weiszfeld (1937). **Sur le point par lequel la somme des distances de  $n$  points donnees est minimum.** *Tohoku Mathematical Journal*.

Compute new weights  $\beta_{i,t} = \frac{1}{\max\{\|z_t - w_i\|_2, \nu\}}$  & Reweighted average  $z_{t+1} = \frac{\sum_i \beta_{i,t} w_i}{\sum_i \beta_{i,t}}$



2. **Clients** compute new weights



# Smoothed Weiszfeld Algorithm

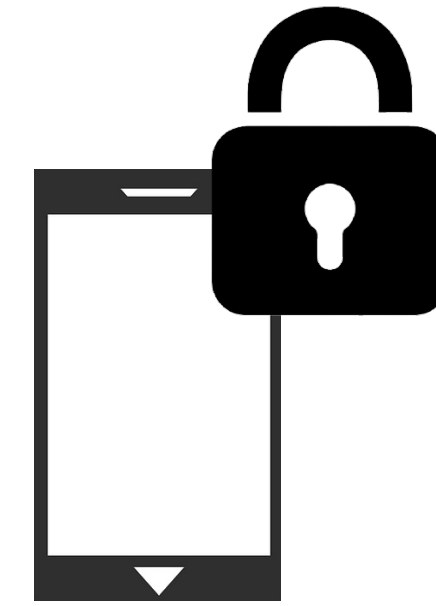
Weiszfeld (1937). **Sur le point par lequel la somme des distances de  $n$  points donnees est minimum.** *Tohoku Mathematical Journal*.

Compute new weights  $\beta_{i,t} = \frac{1}{\max\{\|z_t - w_i\|_2, \nu\}}$  & Reweighted average  $z_{t+1} = \frac{\sum_i \beta_{i,t} w_i}{\sum_i \beta_{i,t}}$

3. Obtain new estimate by **secure averaging**





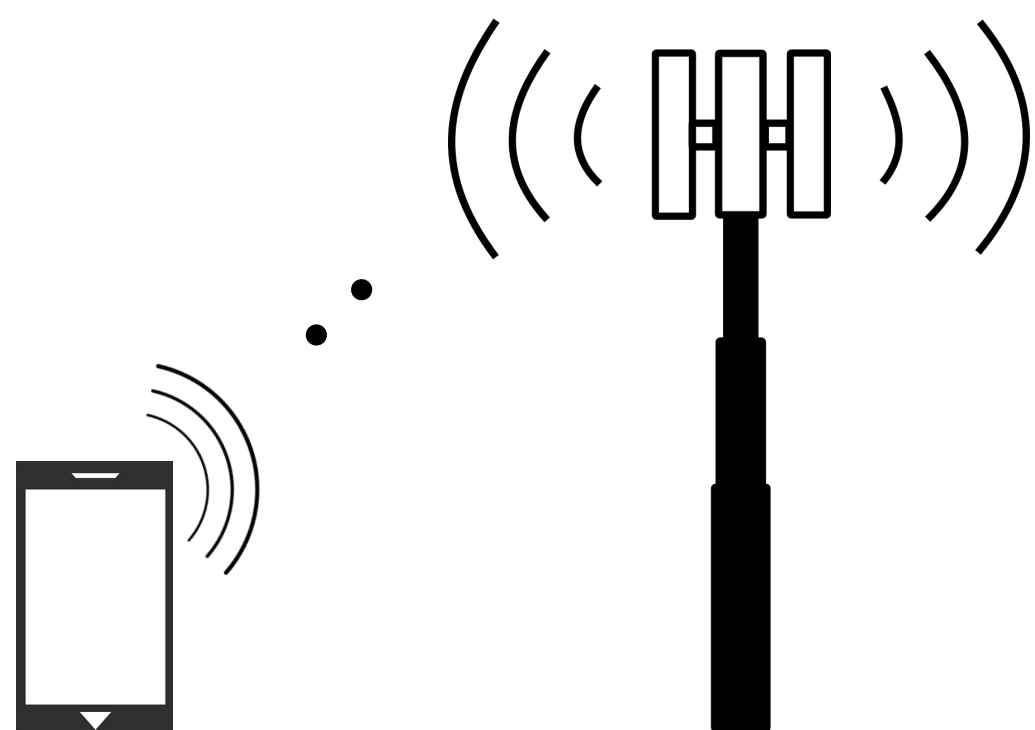


## Secure aggregation

Only client-server communication  
is via **secure average** in the  
**Smoothed Weiszfeld Algorithm**

$$z_{t+1} = \frac{\sum_i \beta_{i,t} w_i}{\sum_i \beta_{i,t}}$$

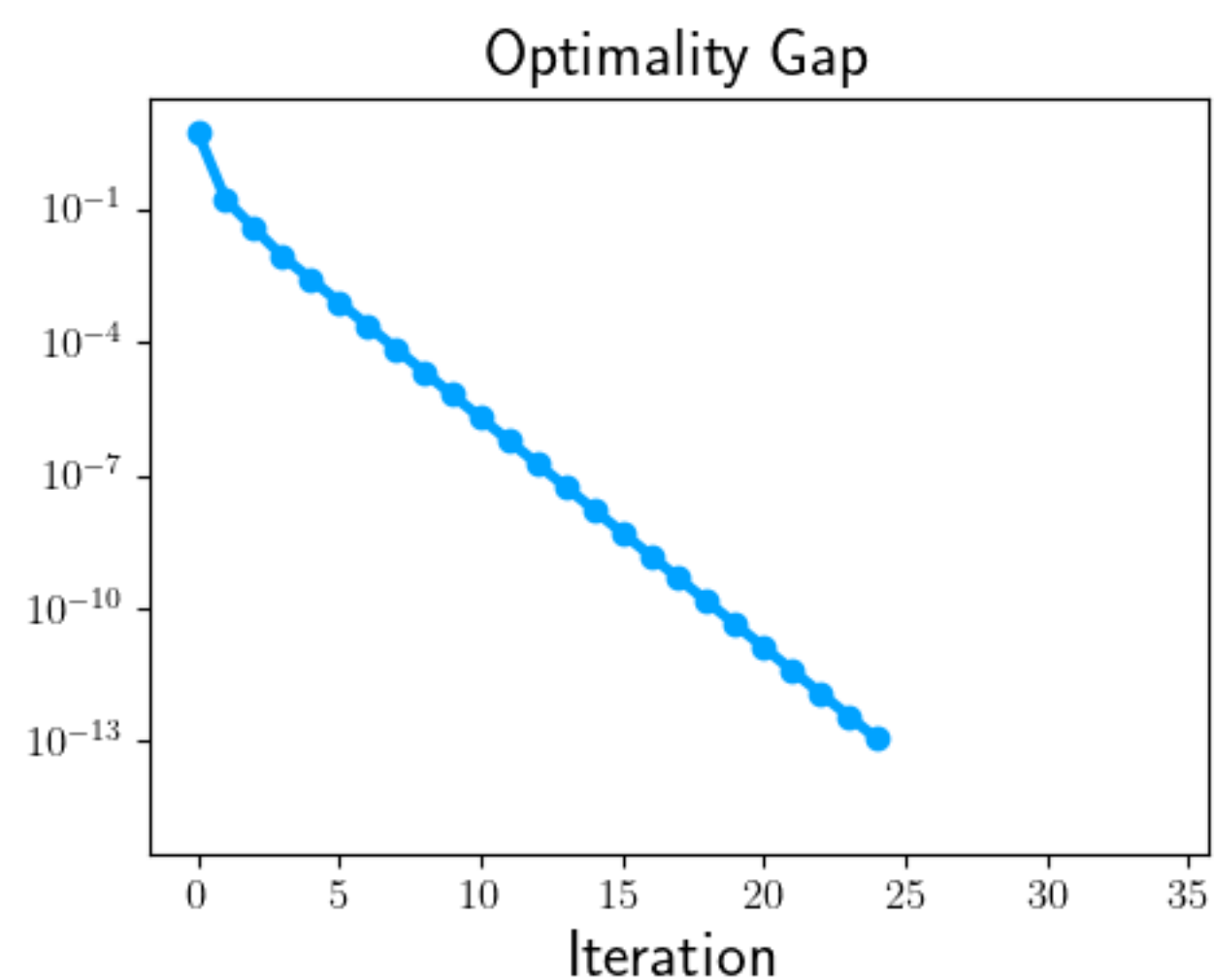




**Communication  
efficient!**

Empirically, **3-5** iterations suffice:

provably rapid convergence

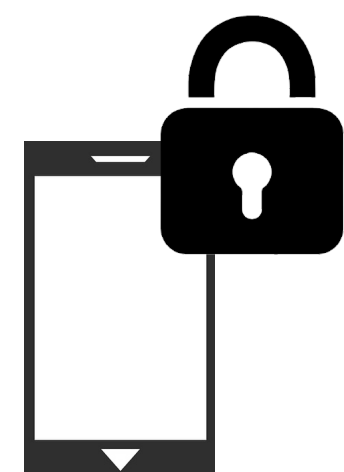
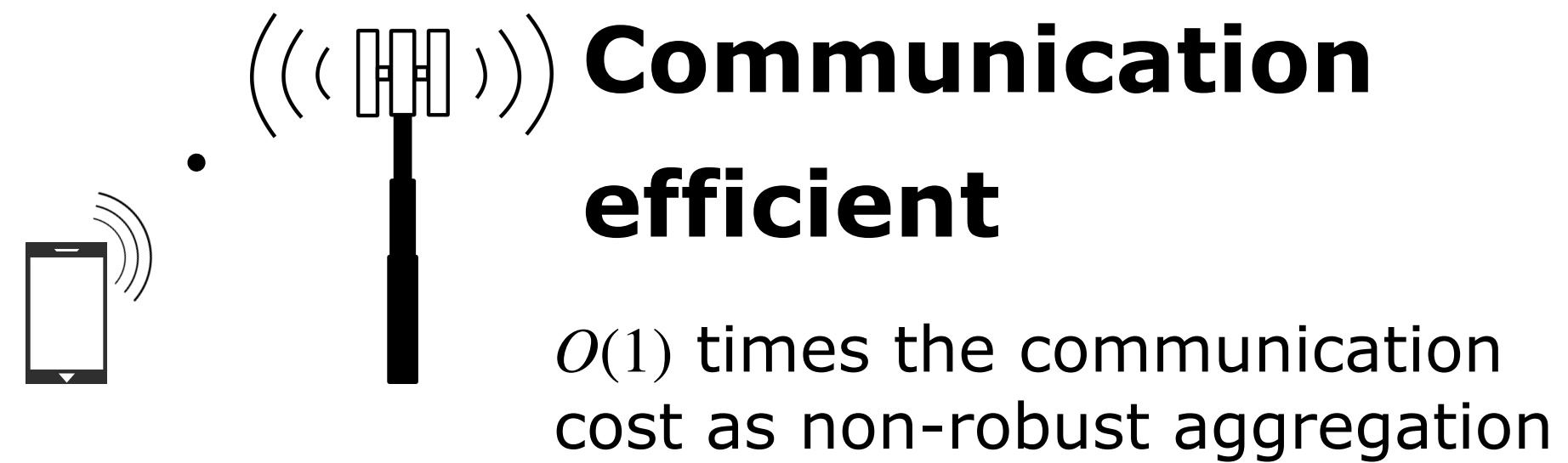
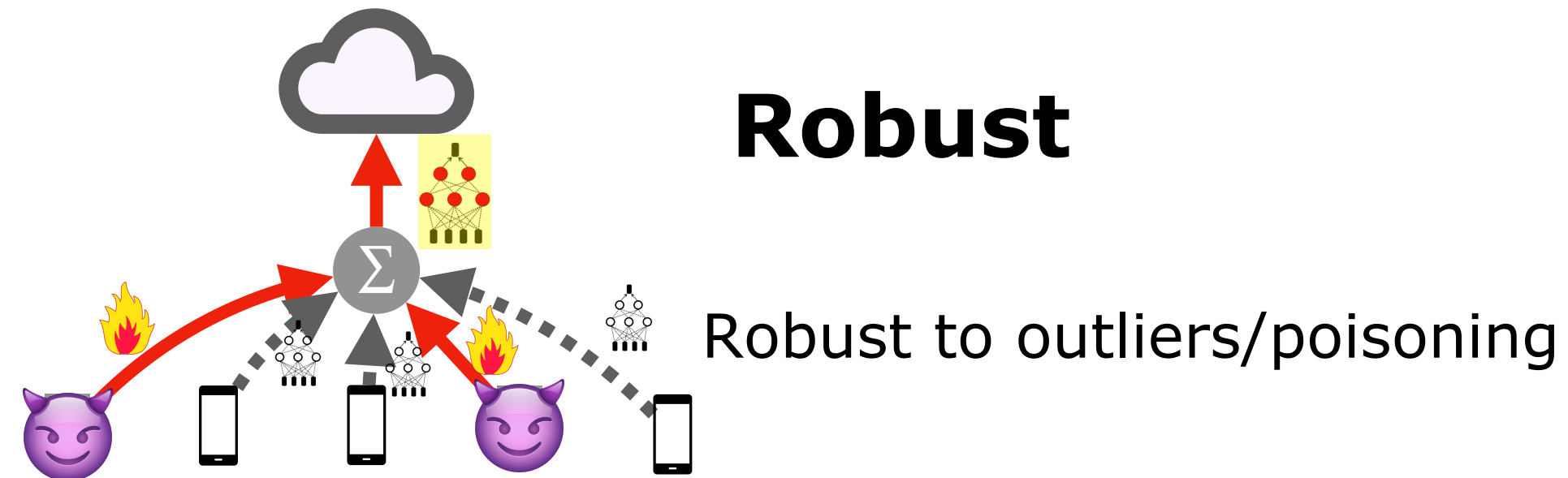


Even **1** iteration  
improves  
robustness!



# *Usual approach* (Direct)

# *Our approach* (Variational)



## **Secure aggregation**

Individual updates not revealed



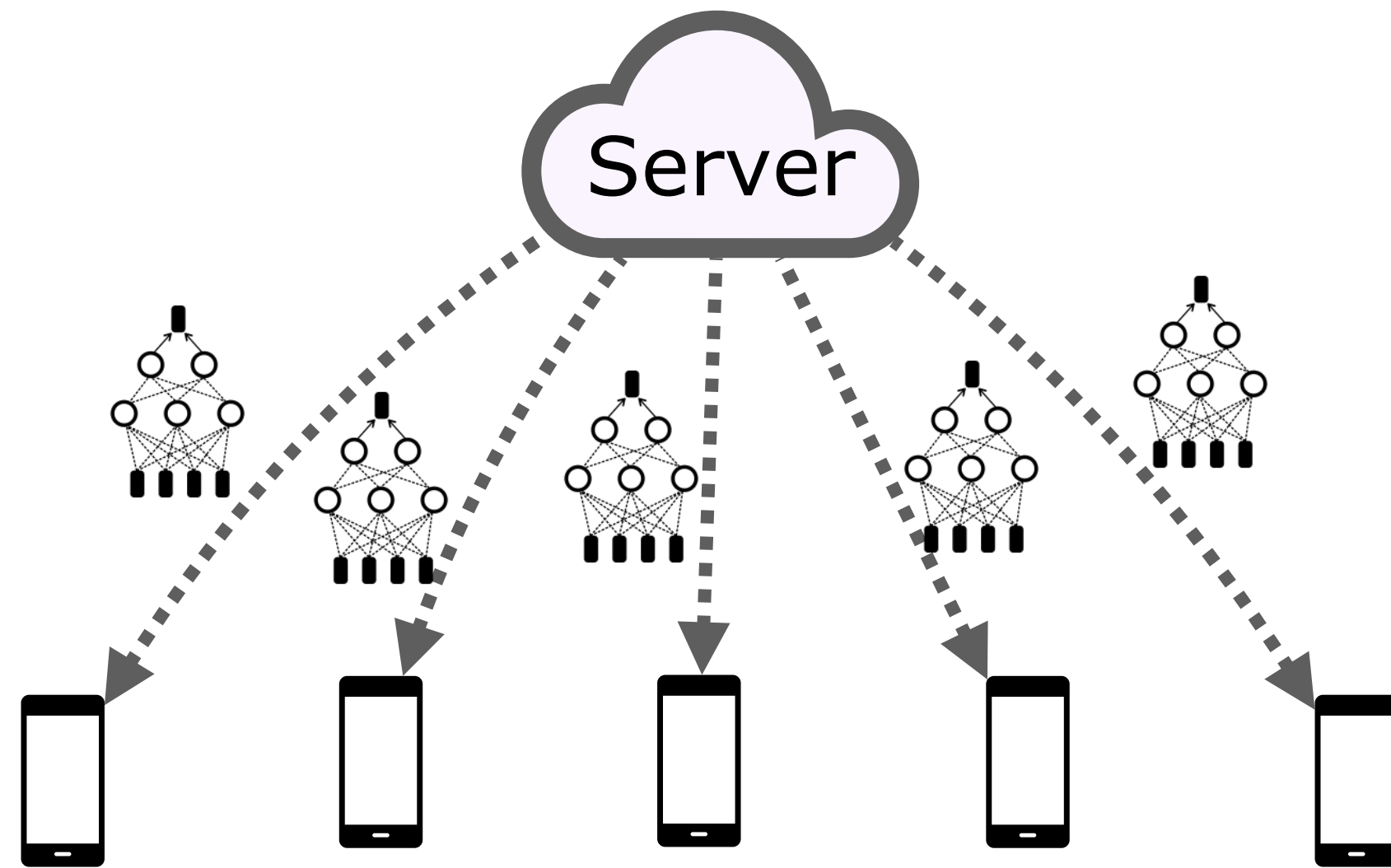


# Robust Federated Aggregation (RFA)

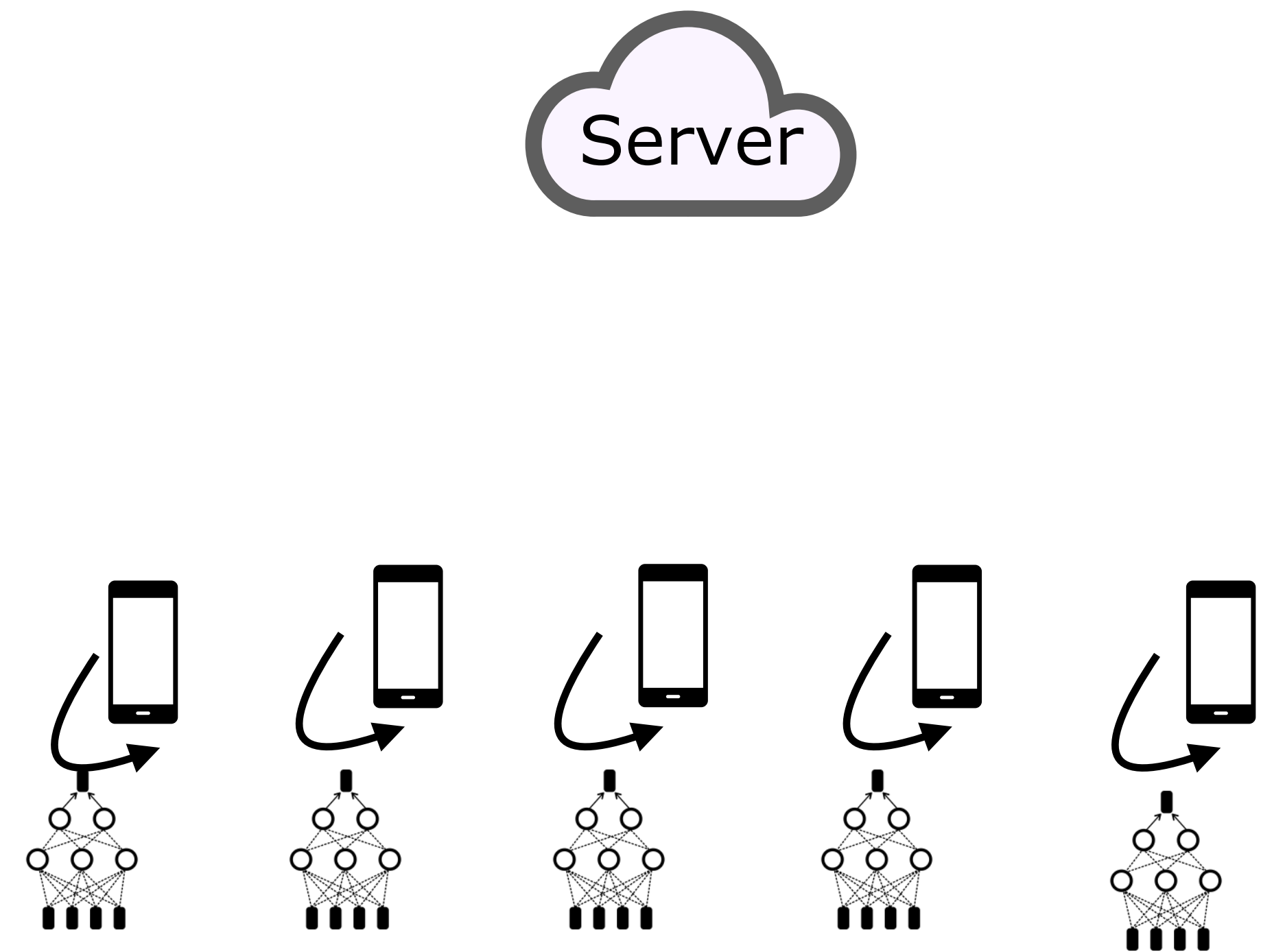
**More robust federated learning =**  
Local SGD steps +  
Geometric median + secure aggregation



*Step 1 of 3: Server broadcasts global model to sampled clients*



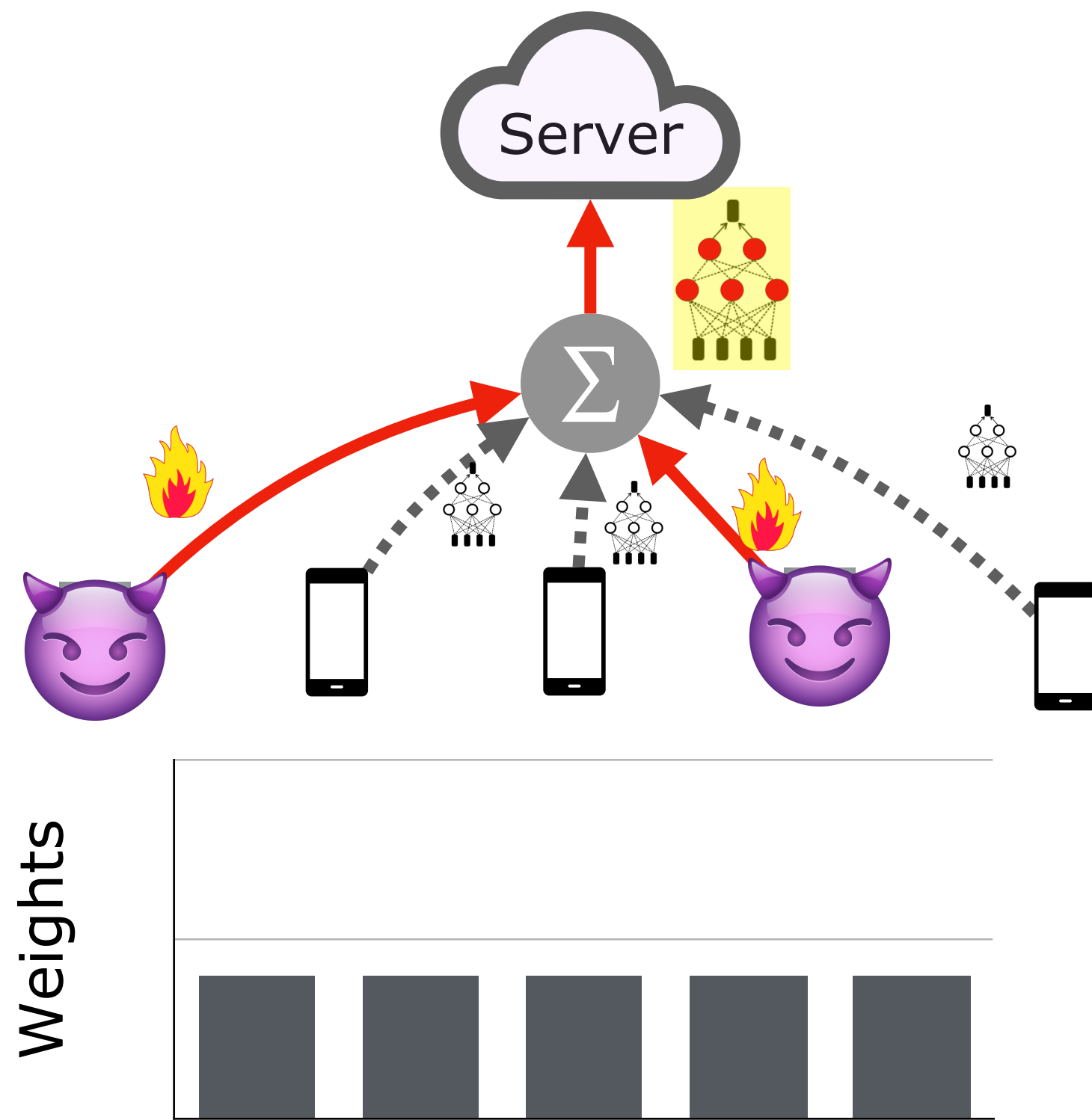
*Step 2 of 3: Clients perform some local SGD steps on their local data*



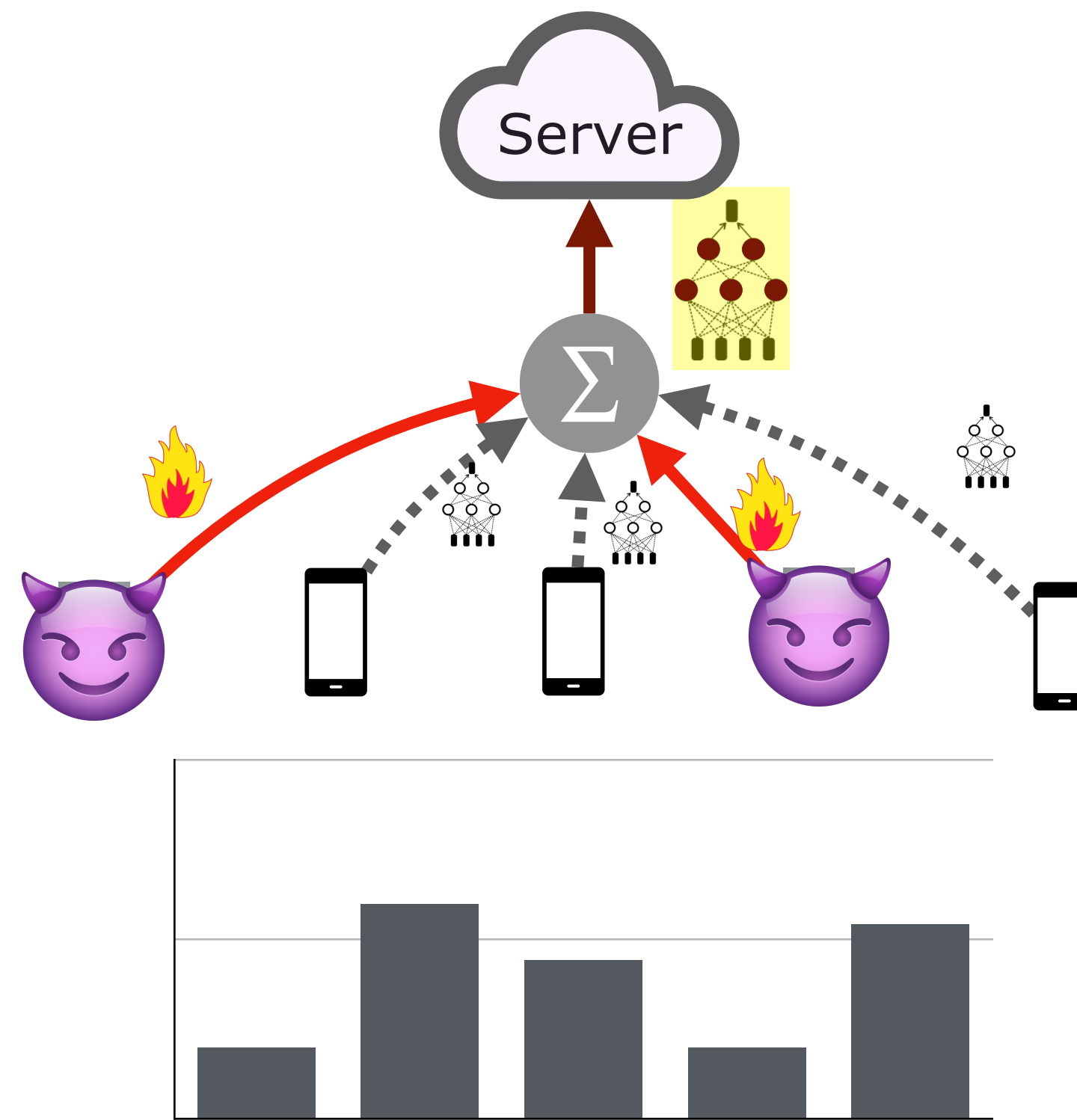
So far, same as federated averaging

*Step 3 of 3: Aggregate with multiple rounds of secure average  
(weights  $\beta_i$  from the Smoothed Weiszfeld Algorithm)*

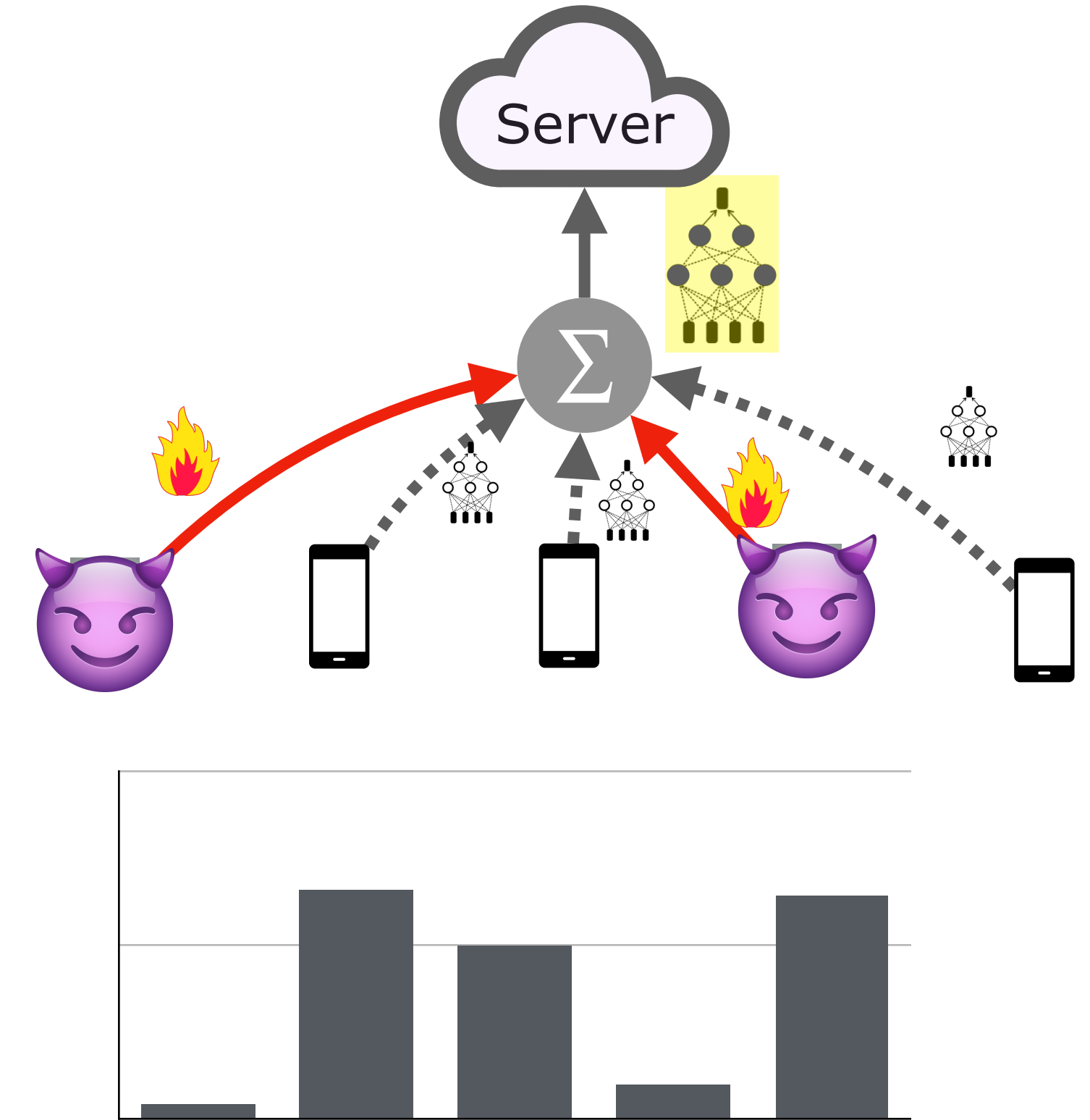
Round 1 of Aggregation



Round 2 of Aggregation



Round 3 of Aggregation





# See the paper for:

IEEE TRANSACTIONS ON  
**SIGNAL PROCESSING**

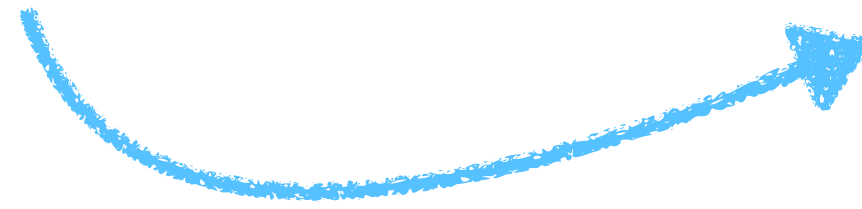
2023

TSP Volume 70 | 2022



## Discussion on *heterogeneity*

## *Convergence* analysis



**The Tension Between Robustness and Heterogeneity:** Heterogeneity is a key property of federated learning. The distribution  $D_i$  of device  $i$  can be quite different from the distribution  $D_j$  of some other device  $j$ , reflecting the heterogeneous data generated by a diverse set of users.

To analyze the effect of heterogeneity on robustness, consider the simplified scenario of robust mean estimation in Huber's contamination model [34]. Here, we wish to estimate the mean  $\mu \in \mathbb{R}^d$  given samples  $w_1, \dots, w_m \sim (1 - \rho)\mathcal{N}(\mu, \sigma^2 I) + \rho Q$ , where  $Q$  denotes some outlier distribution that  $\rho$ -fraction of the points (designated as outliers) are drawn from. Any aggregate  $\bar{w}$  must satisfy the lower bound  $\|\bar{w} - \mu\|^2 \geq \Omega(\sigma^2 \max\{\rho^2, d/m\})$  with constant probability [69, Theorem 2.2]. In the federated learning setting, more heterogeneity corresponds to a greater variance  $\sigma^2$  among the inlier points, implying a larger error in mean estimation. This suggests a tension between robustness and heterogeneity, where increasing heterogeneity makes robust mean estimation harder in terms of  $\ell_2$  error.

In this work, we strike a compromise between robustness and heterogeneity by considering a family  $\mathcal{D}$  of allowed data

**Convergence:** We now analyze RFA where the local SGD updates are equipped with “tail-averaging” [73] so that  $w_i^{(t+1)} = (2/\tau) \sum_{k=\tau/2}^{\tau} w_{i,k}^{(t)}$  is averaged over the latter half of the trajectory of iterates instead of line 9 of Algorithm 1. We show that this variant of RFA converges up to the dissimilarity level  $\Omega = \Omega_X \Omega_{Y|X}$  when the corruption level  $\rho < 1/2$ .

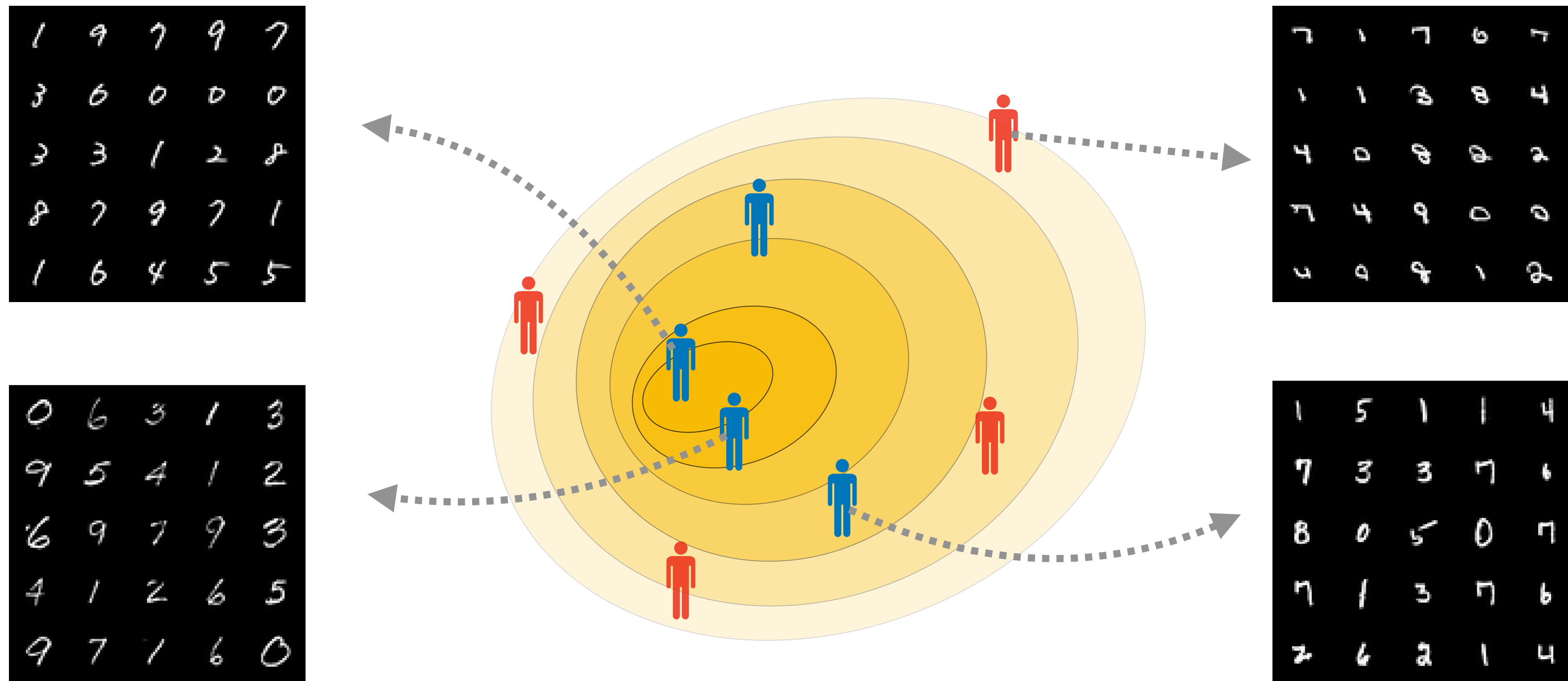
**Theorem 4:** Consider  $F$  defined in (7) and suppose the corruption level satisfies  $\rho < 1/2$ . Consider Algorithm 1 run for  $T$  outer iterations with a learning rate  $\gamma = 1/(2R^2)$ , and the local updates are run for  $\tau_t$  steps in outer iteration  $t$  with tail averaging. Fix  $\delta > 0$  and  $\theta \in (\rho, 1/2)$ , and set the number of devices per iteration,  $m$  as

$$m \geq \frac{\log(T/\delta)}{2(\theta - \rho)^2}. \quad (11)$$

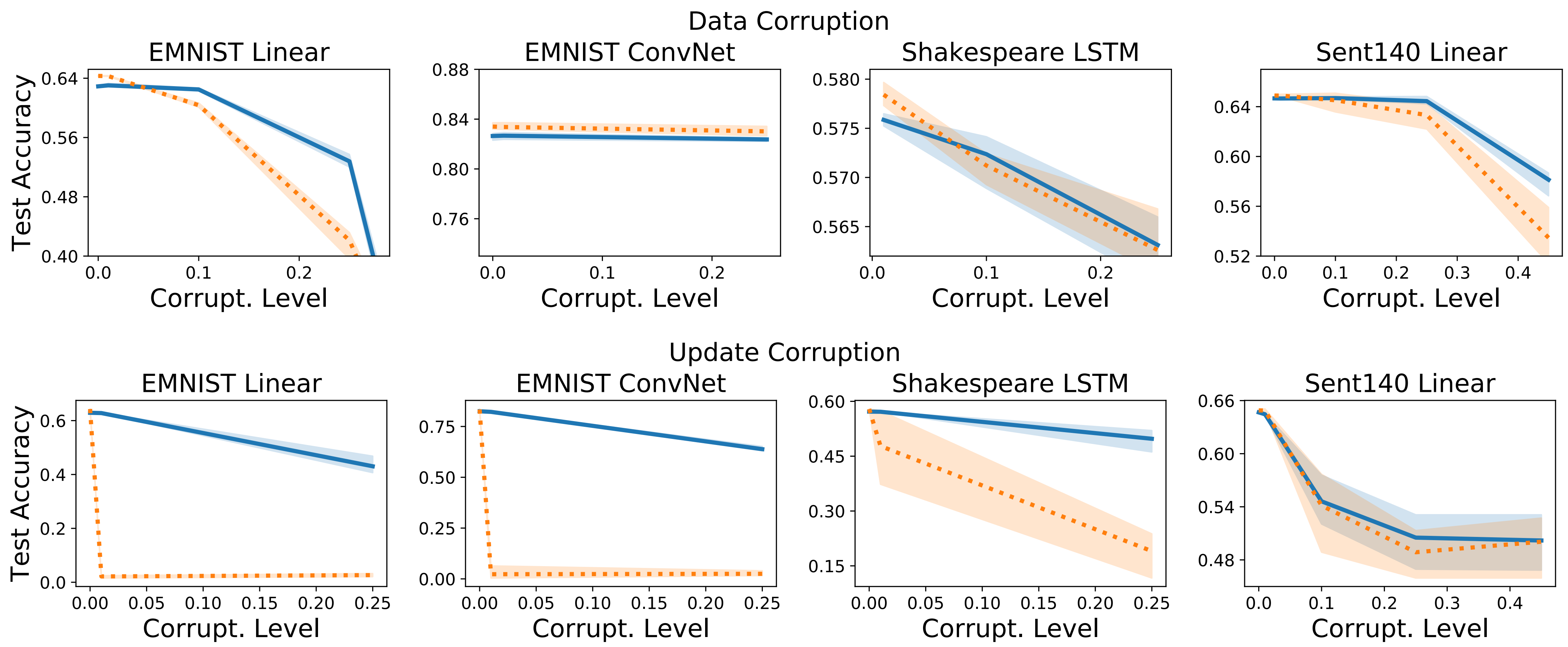
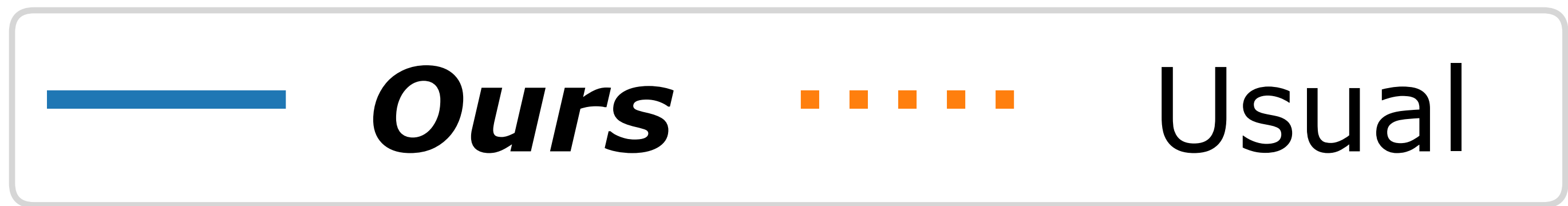
Define  $C_\theta := (1 - 2\theta)^{-2}$ ,  $w^\star = \arg \min F$ ,  $F^\star = F(w^\star)$ ,  $\kappa := R^2/\mu$  and  $\Delta_0 := \|w^{(0)} - w^\star\|^2$ . Let  $\tau \geq 4\kappa \log(128C_\theta\kappa)$ . We have that the event  $\mathcal{E} = \bigcap_{t=0}^{T-1} \{|S_t \cap \mathcal{C}| \leq \theta m\}$  holds with probability at least  $1 - \delta$ . Further, if  $\tau_t = 2^t \tau$  for each iteration  $t$ , then the output  $w^{(T)}$  of Algorithm 1 satisfies,

$$\mathbb{E} \left[ \|w^{(T)} - w^\star\|^2 \mid \mathcal{E} \right] \leq \frac{\Delta_0}{2^T} + CC_\theta \left( \frac{d\sigma^2 T}{\mu\tau 2^T} + \frac{\epsilon^2}{m^2} + \Omega^2 \right)$$

# Experiments and Improvements







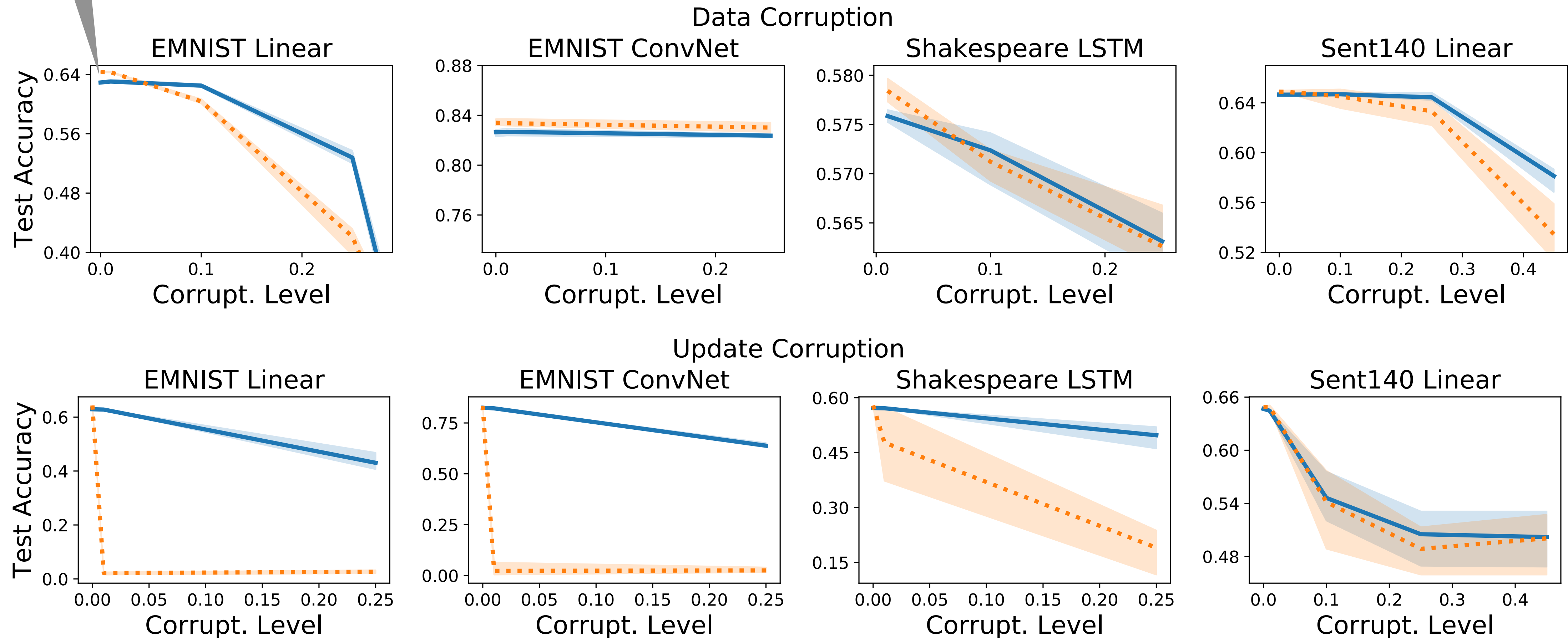
1.4pp  
gap at zero  
corruption



***Ours***



**Usual**

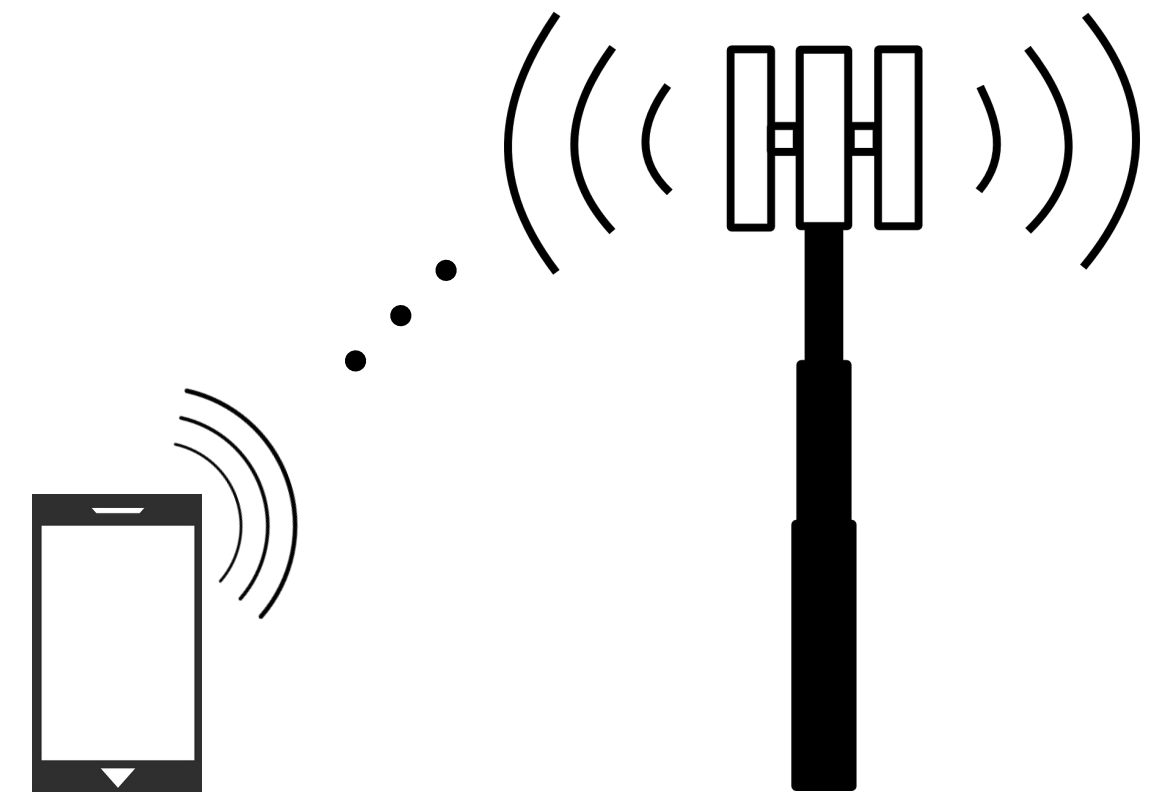


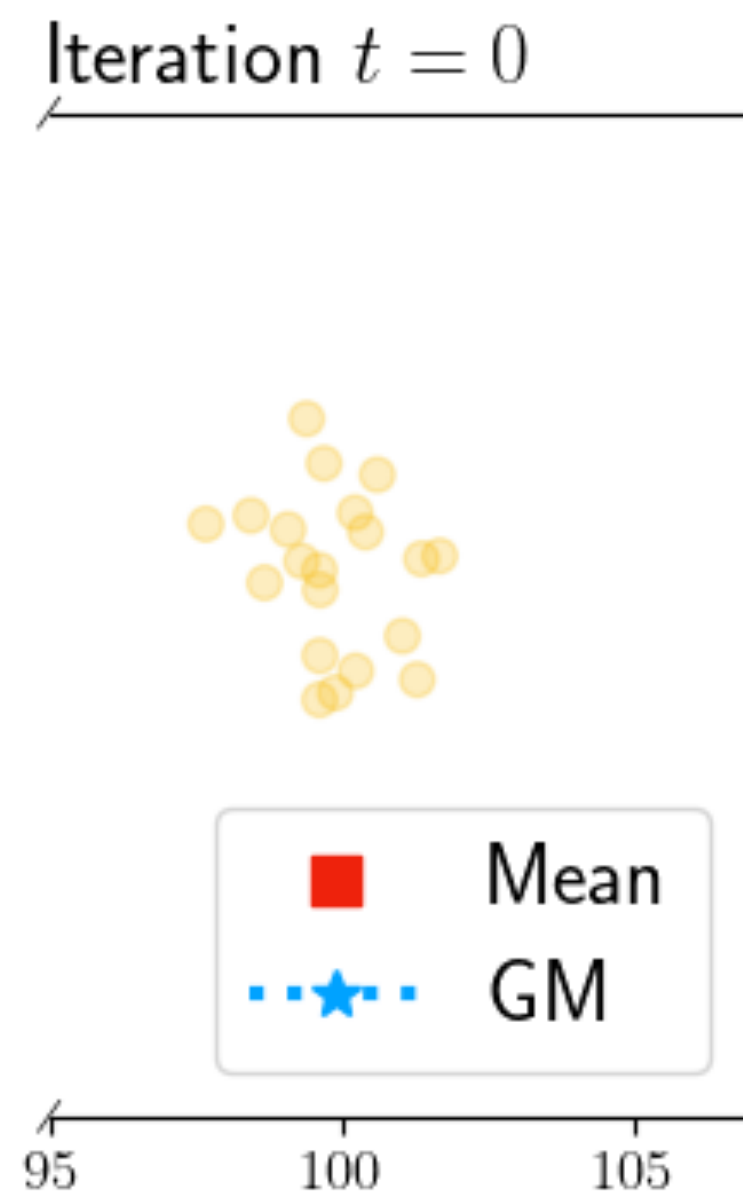
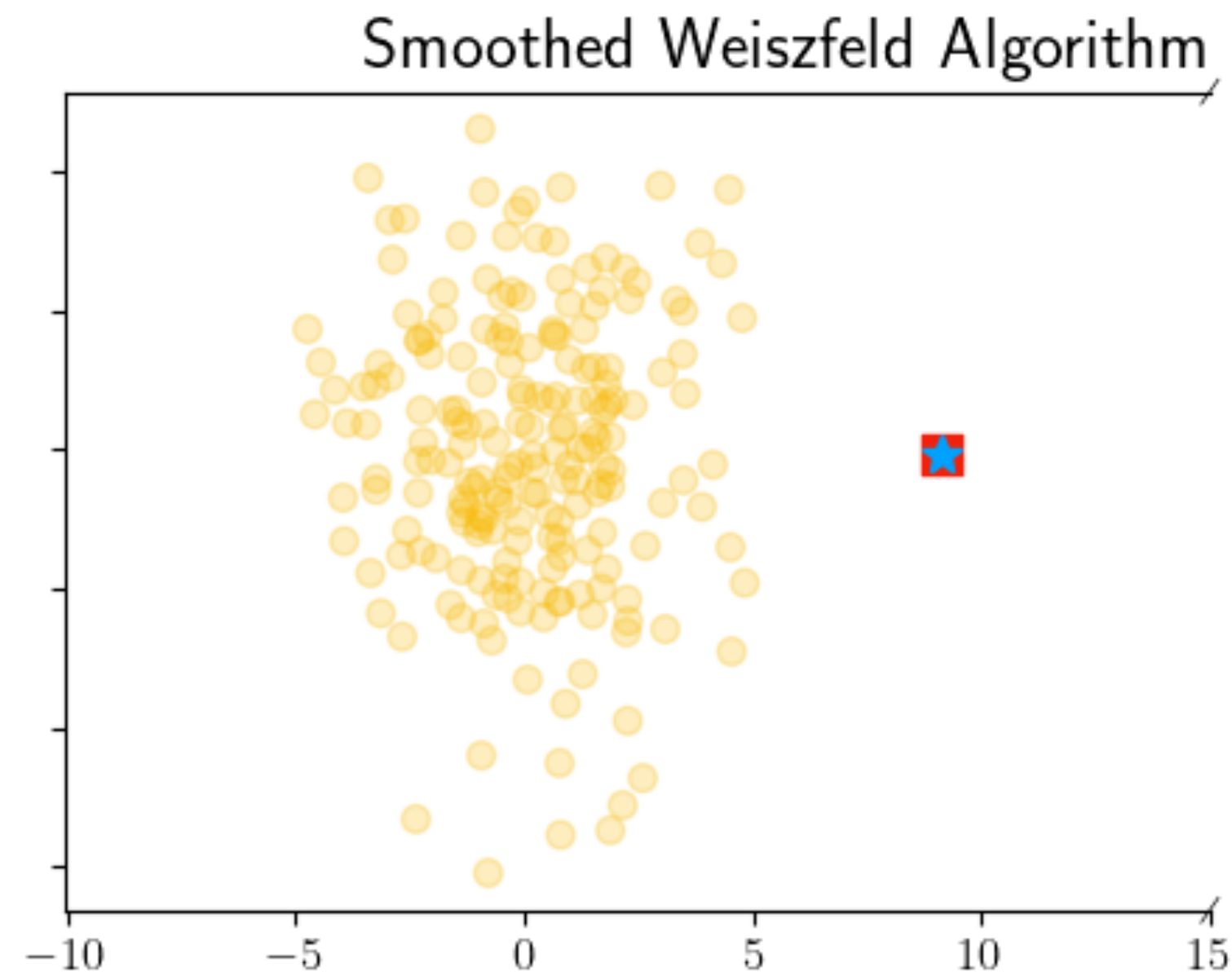


# Reducing the communication cost

One round of our algorithm  $\Rightarrow$  3-5 rounds of communication

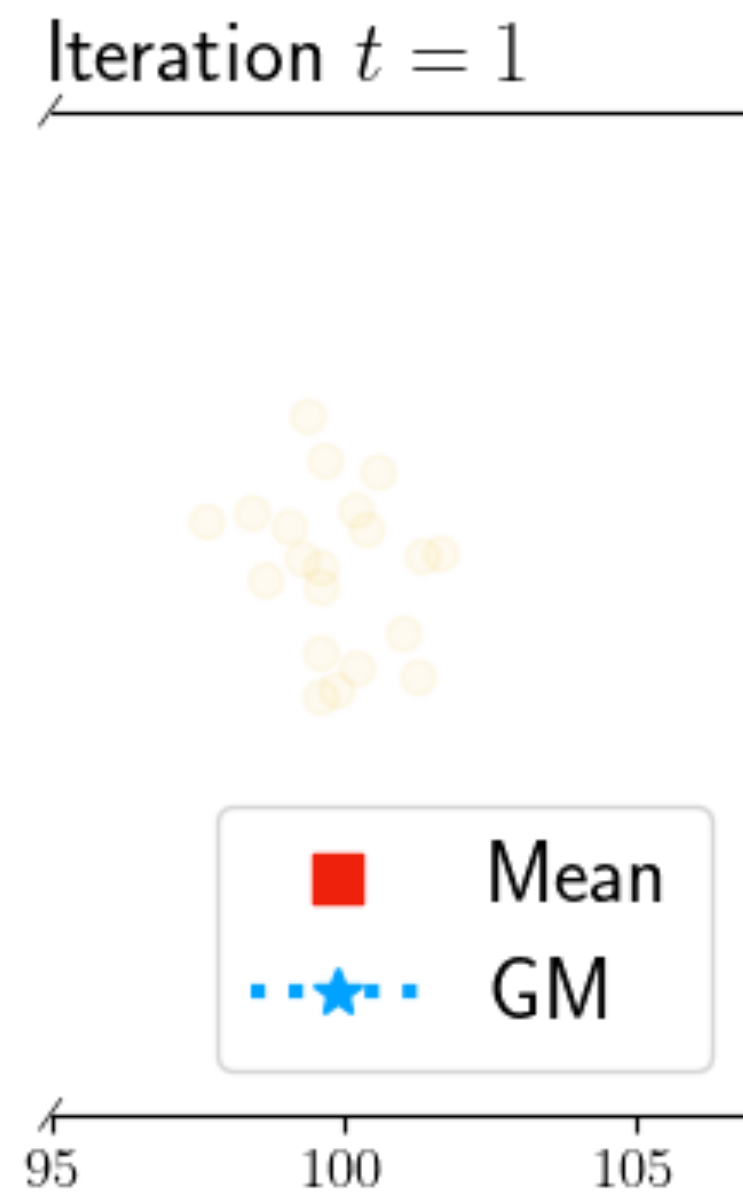
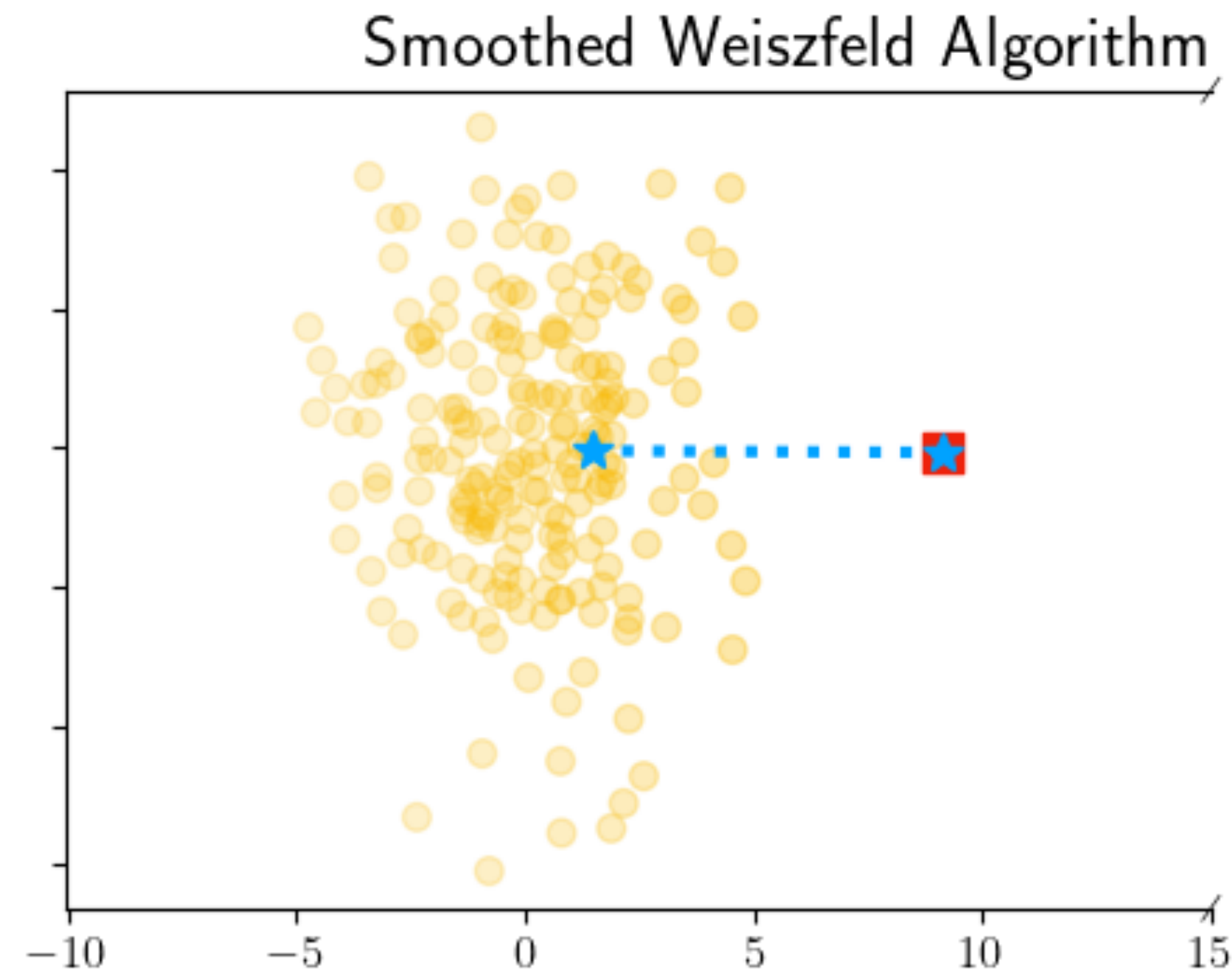
Due to iterations of the smoothed Weiszfeld algorithm





Does 1 round of communication improve robustness?

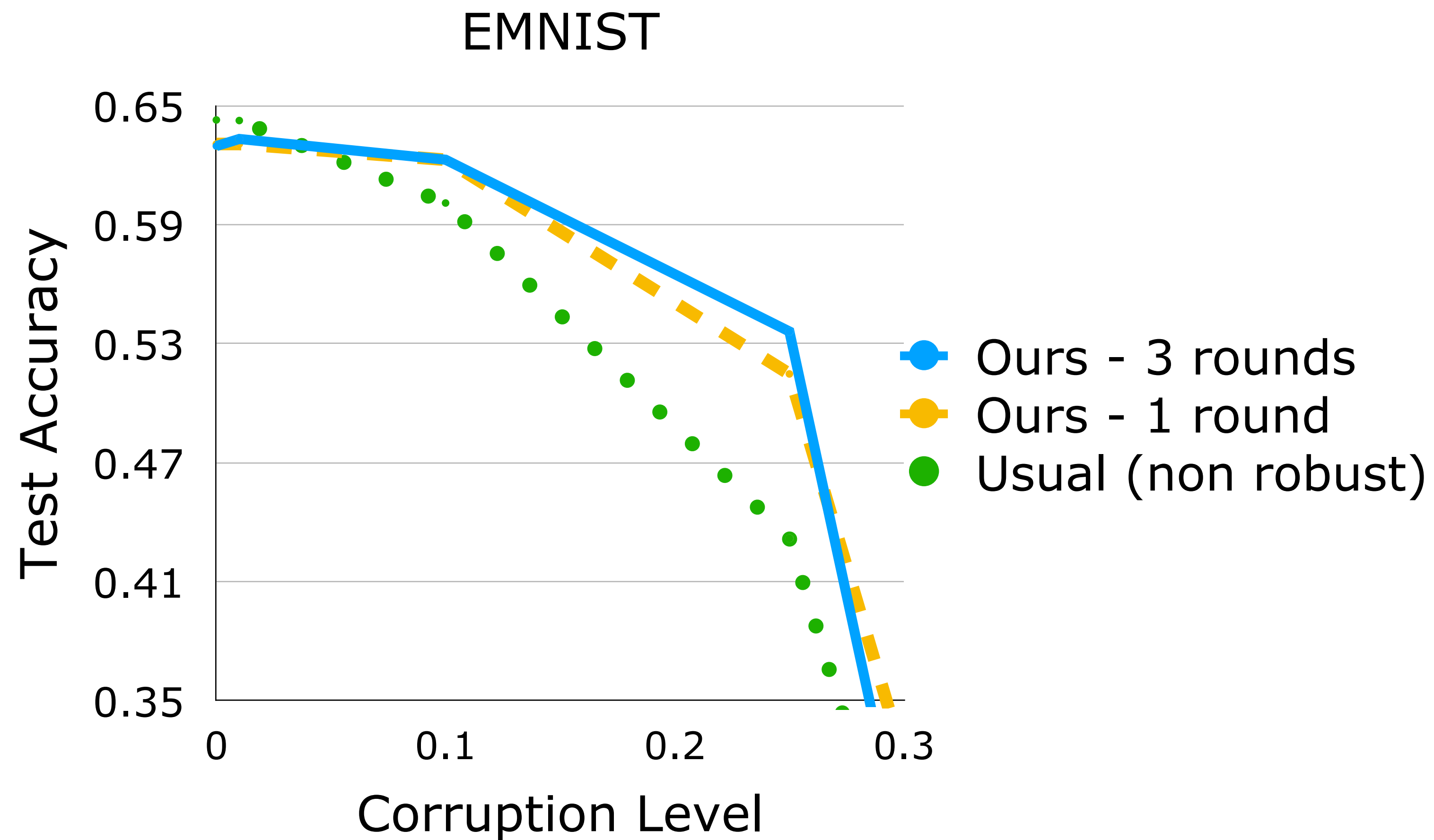
$$\beta_i = \frac{1}{\max\{\|w_i\|_2, \nu\}}$$



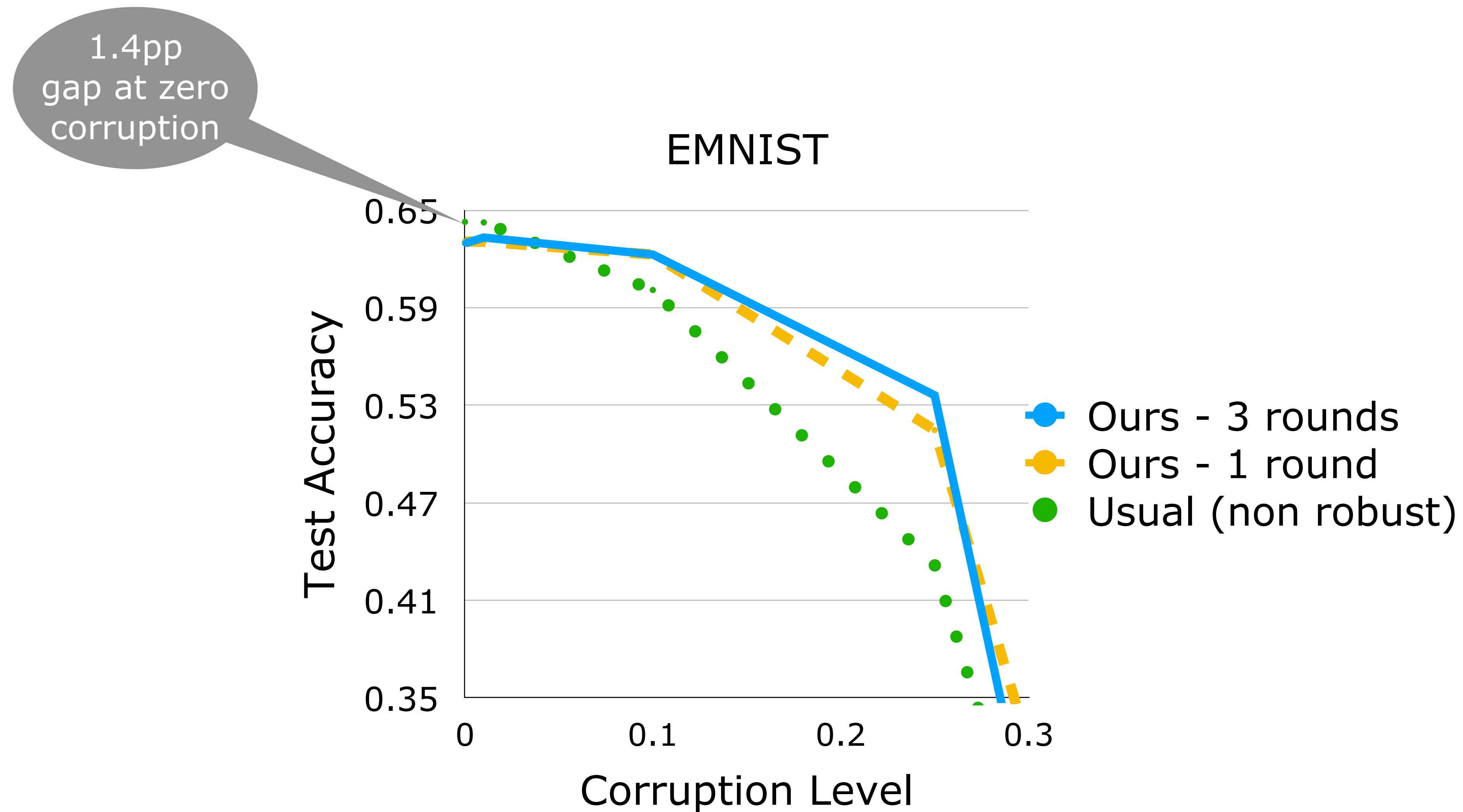
$$z = \frac{\sum_i \beta_i w_i}{\sum_i \beta_i}$$



# ***1 communication round*** already improves robustness



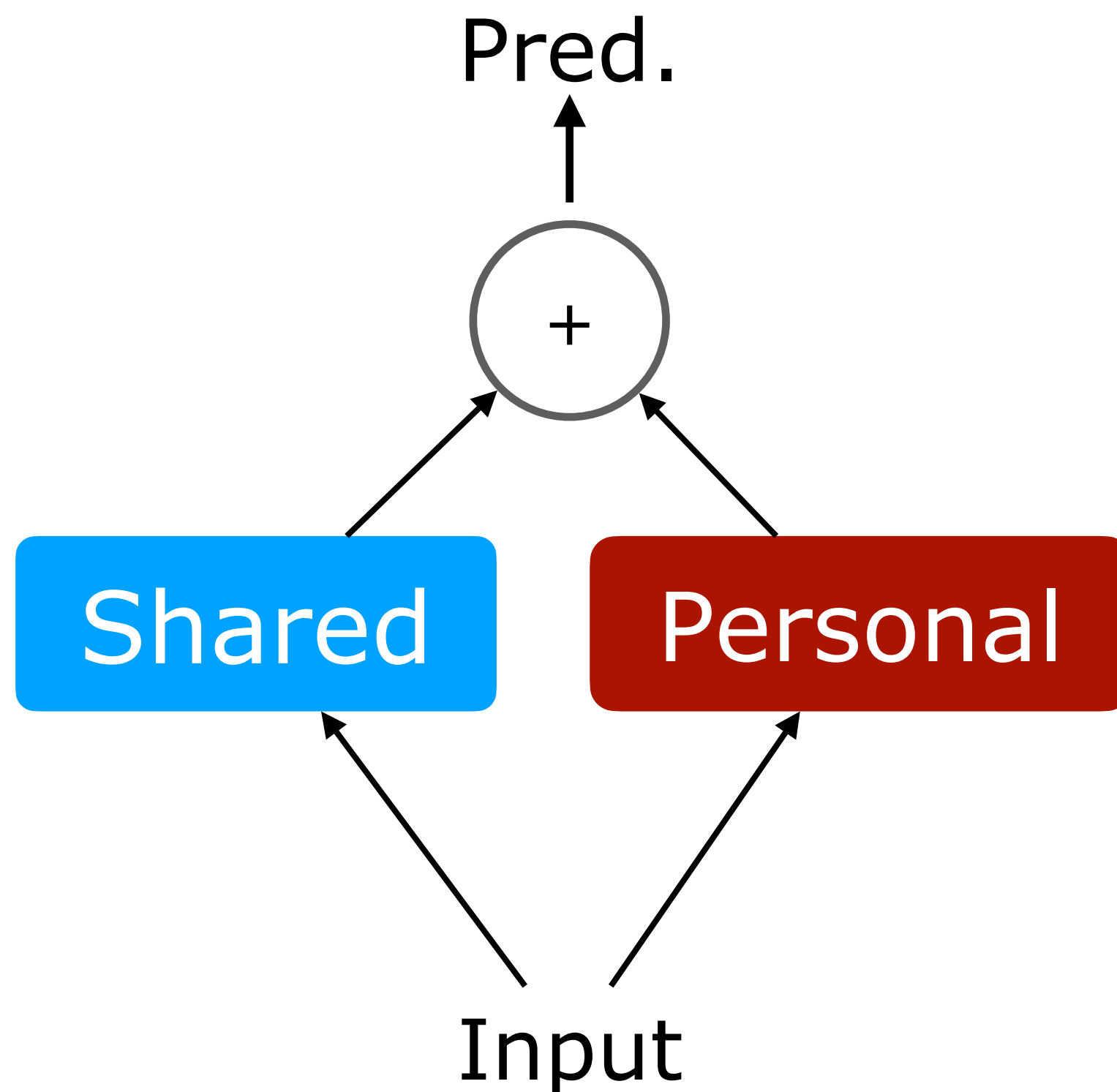
# How do we get rid of this gap?





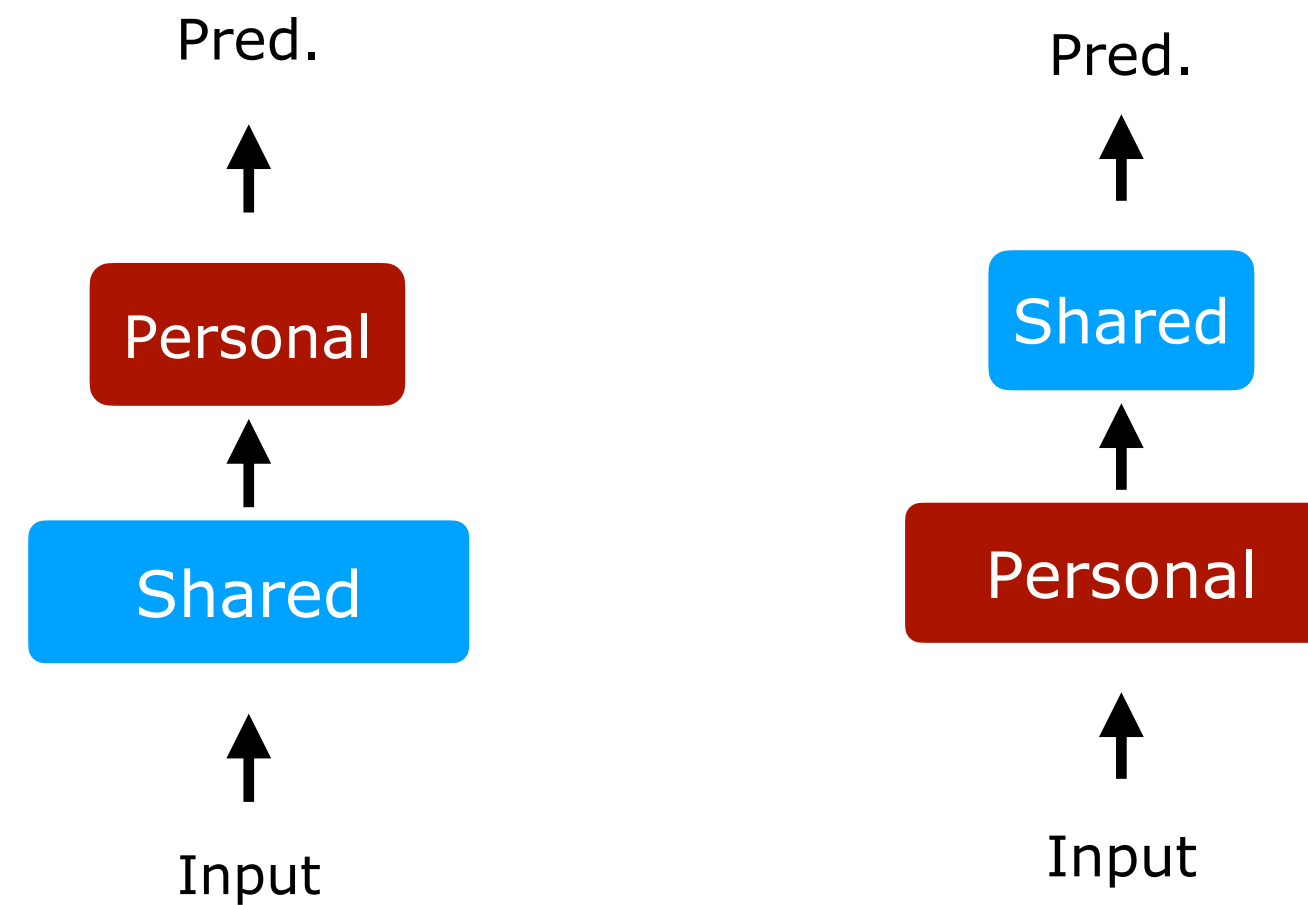
# Model personalization

The model has a global component  
and a per-client component



$$\begin{array}{l} \text{Shared Params } u \\ + \text{ Personal Params } v_i \end{array} = \text{Full model } w_i = (u, v_i)$$

# Personalization Architectures



Arivazhagan et al. (2019)  
Collins et al. (2021)

Liang et al. (2019)

**Multi-task learning:** Caruana (1997), Baxter (2000), Evgeniou & Pontil (2004), Collobert & Weston (2005), Argyriou et al. (2008), ...



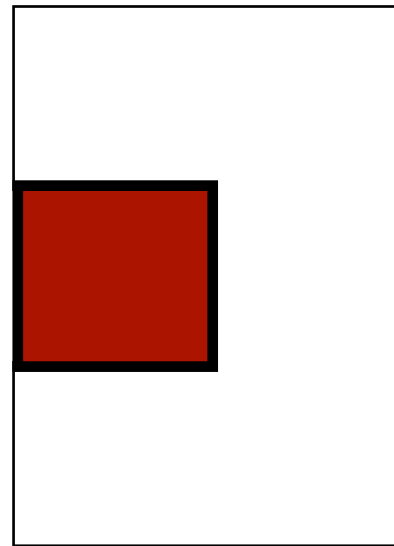
# Optimization

Shared Params  $u$



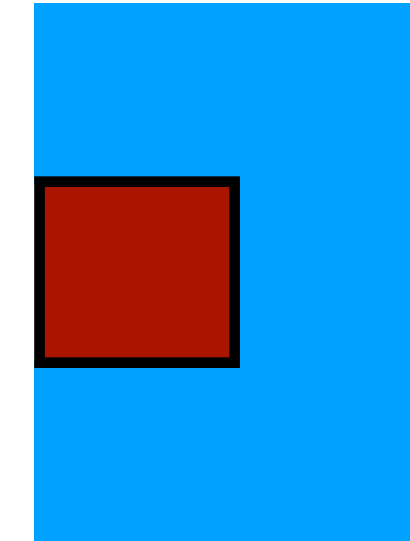
+

Personal Params  $v_i$



=

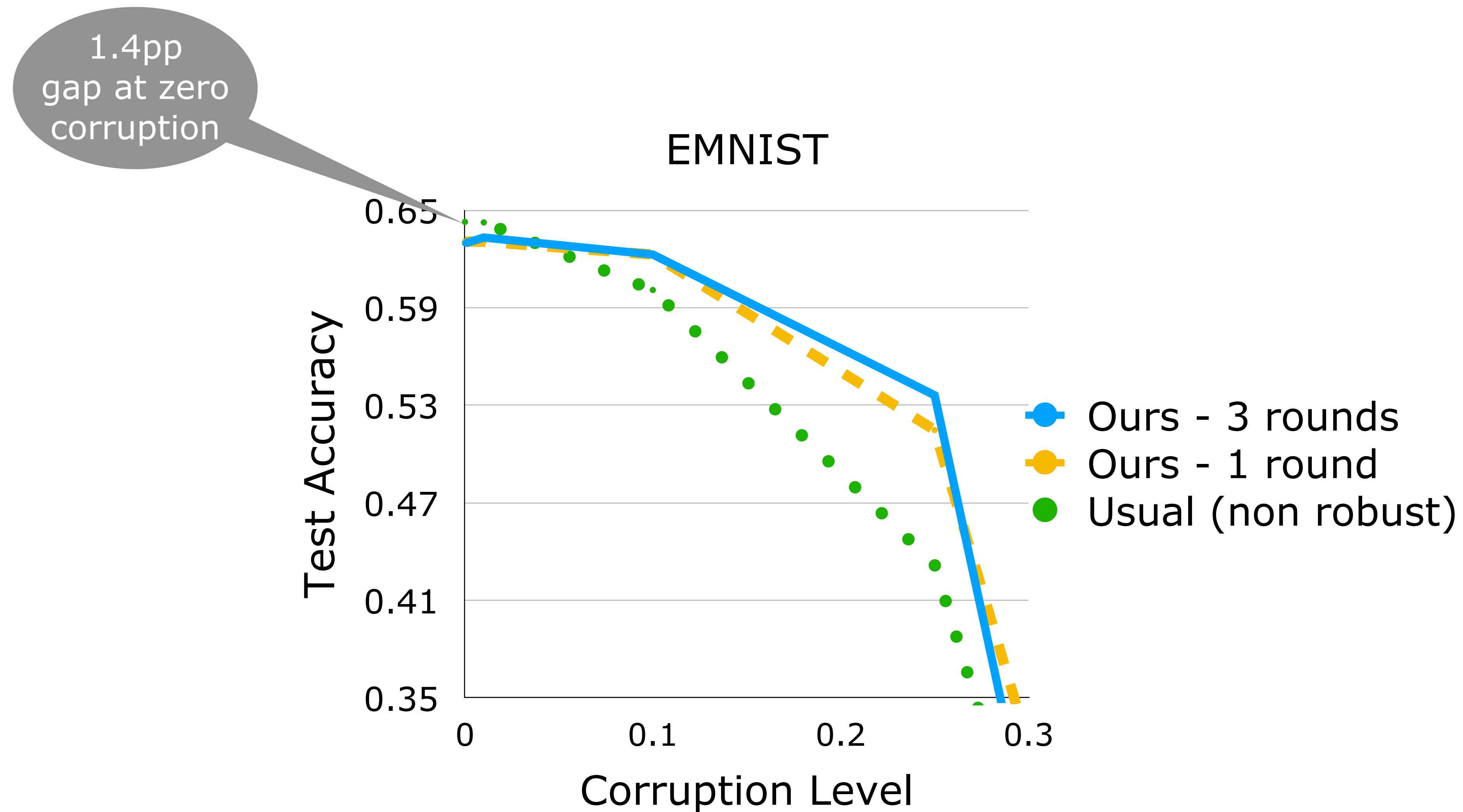
Full model  $w_i = (u, v_i)$



***Shared part of the model*** is  
updated with robust aggregation

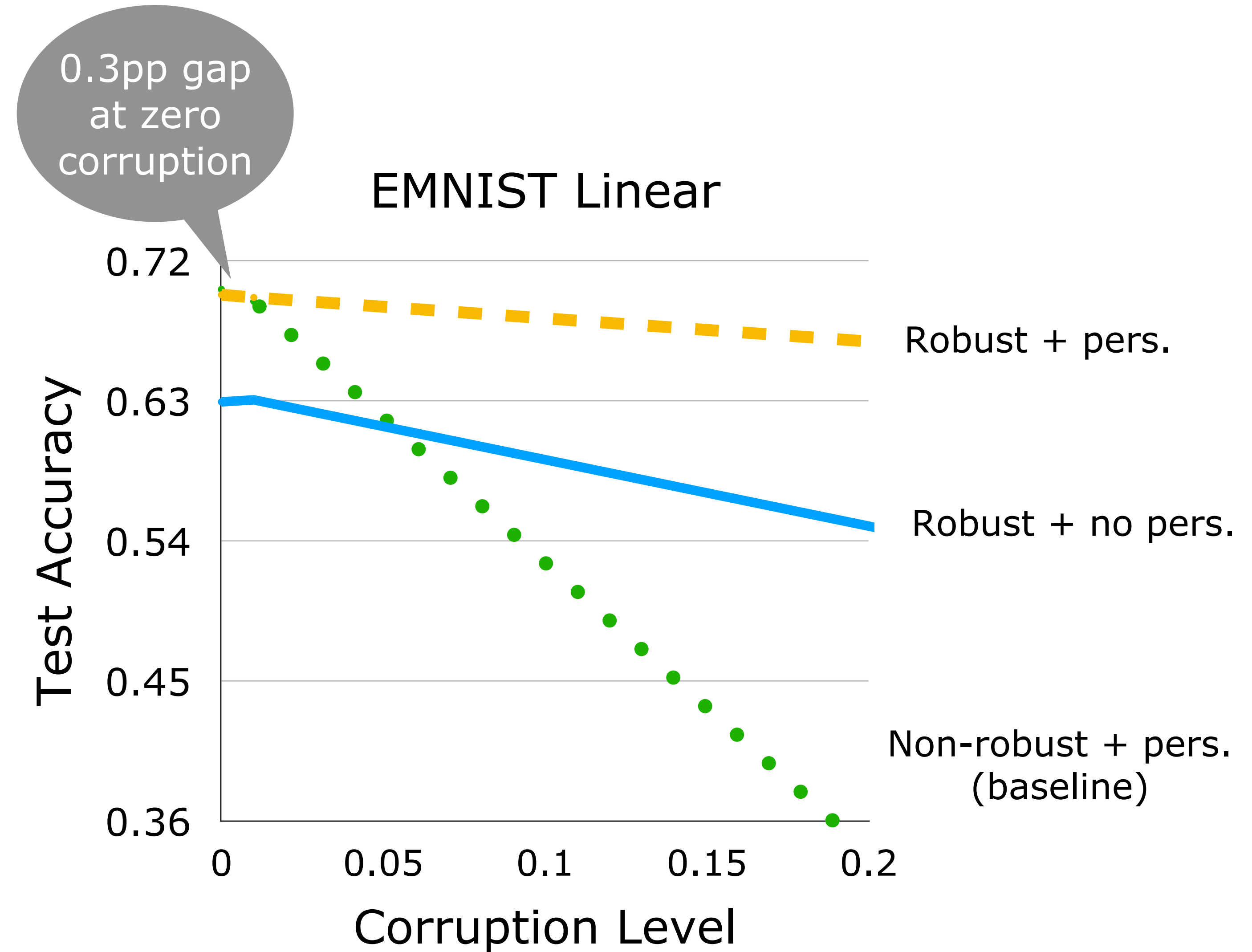
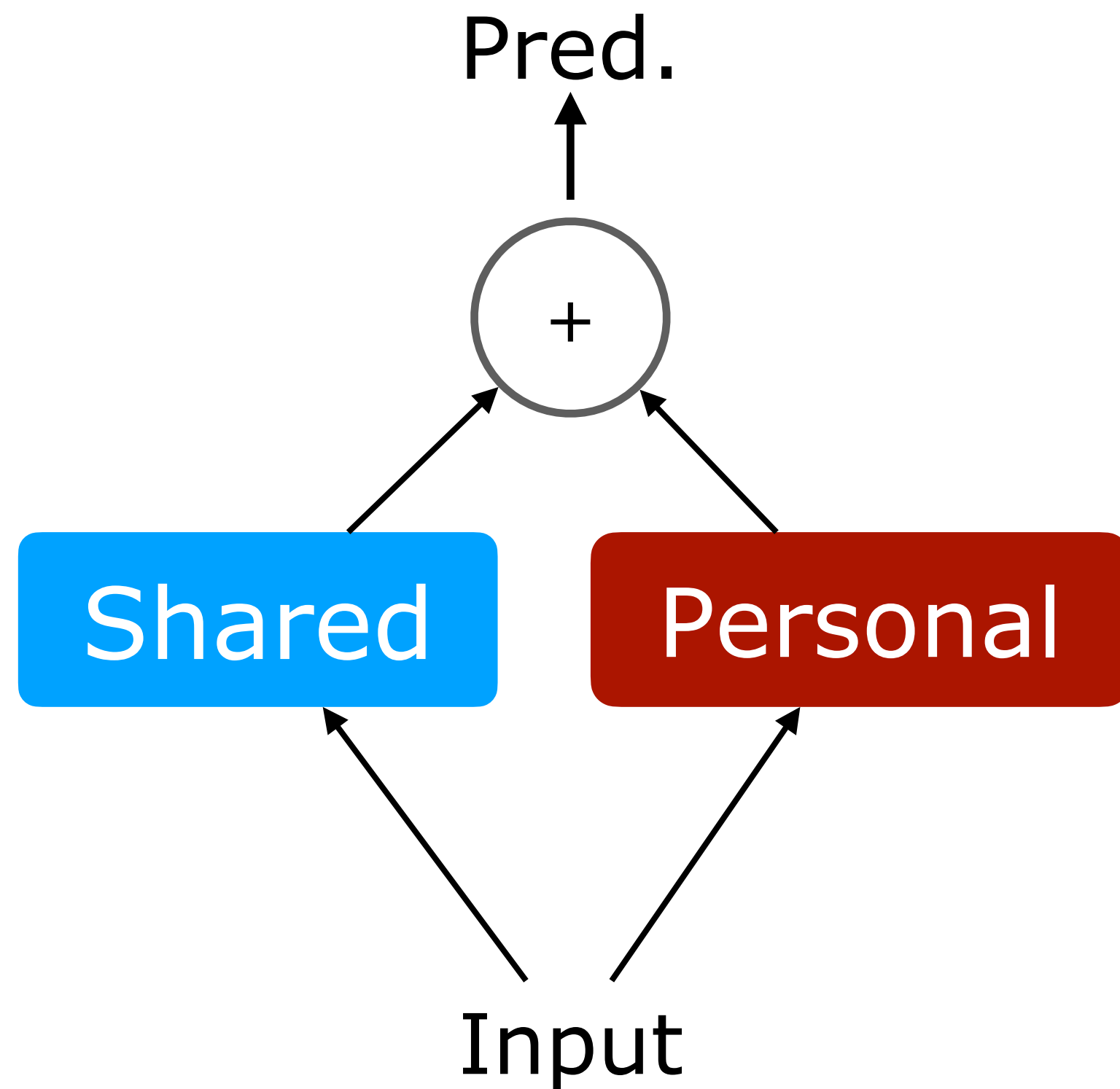
***Personal part of the model***  
stays with the client

# Does personalization get rid of this gap?





# Yes, we can improve robust aggregation with personalization!



***In the literature:***

Robust Federated Aggregation (RFA)



# RFA is certifiably more robust to backdoor attacks

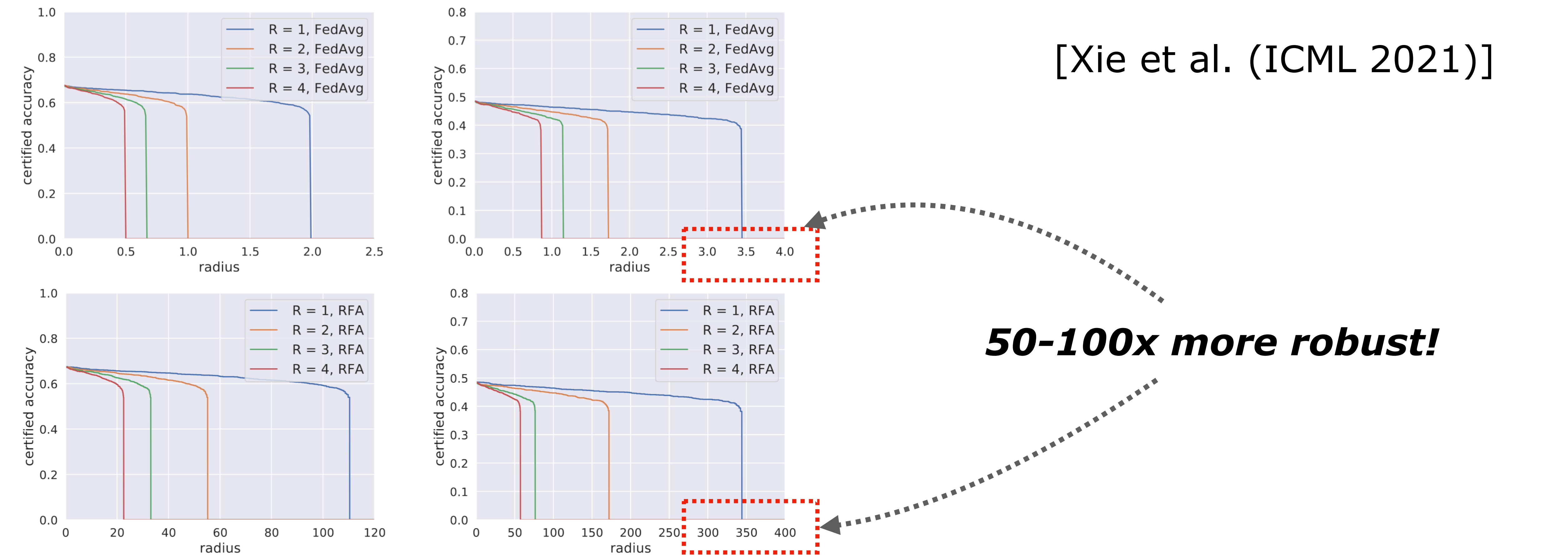


Figure 6. Certified accuracy on MNIST (left) and EMNIST (right) with different  $R$  when FL is trained under the robust aggregation RFA (Pillutla et al., 2019).

Xie, Chen, Chen, Li. **CRFL: Certifiably Robust Federated Learning against Backdoor Attacks.** ICML 2021.

# RFA is asymptotically strategy-proof

**Strategy-proof:** Can a device lie to bring the aggregate to a desired point?

With a large number of independent devices, RFA is approximately strategy-proof

---

## On the Strategyproofness of the Geometric Median

---

**El-Mahdi El-Mhamdi**  
Calicarpa, École Polytechnique

**Sadegh Farhadkhani\***  
EPFL

**Rachid Guerraoui**  
EPFL

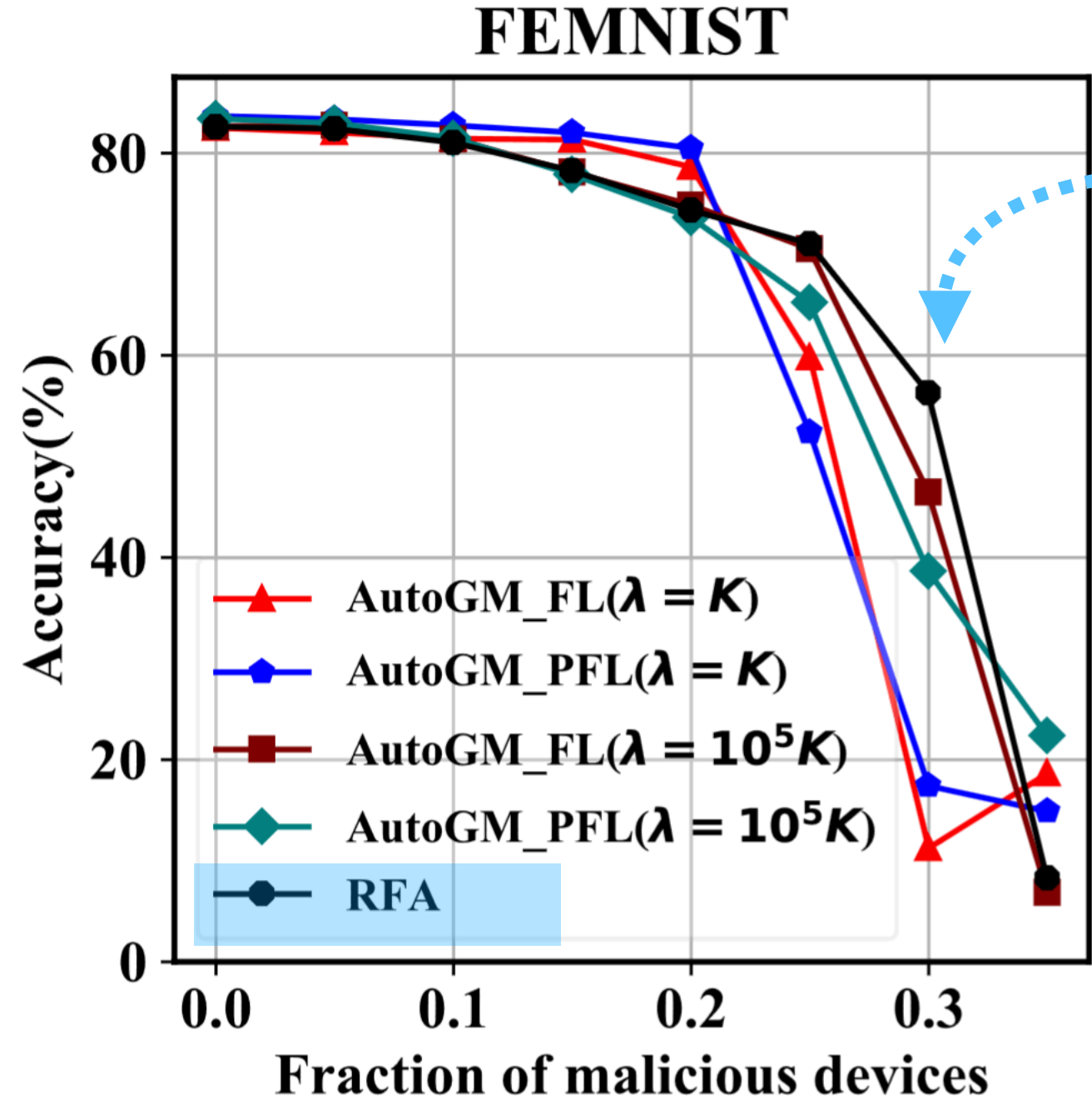
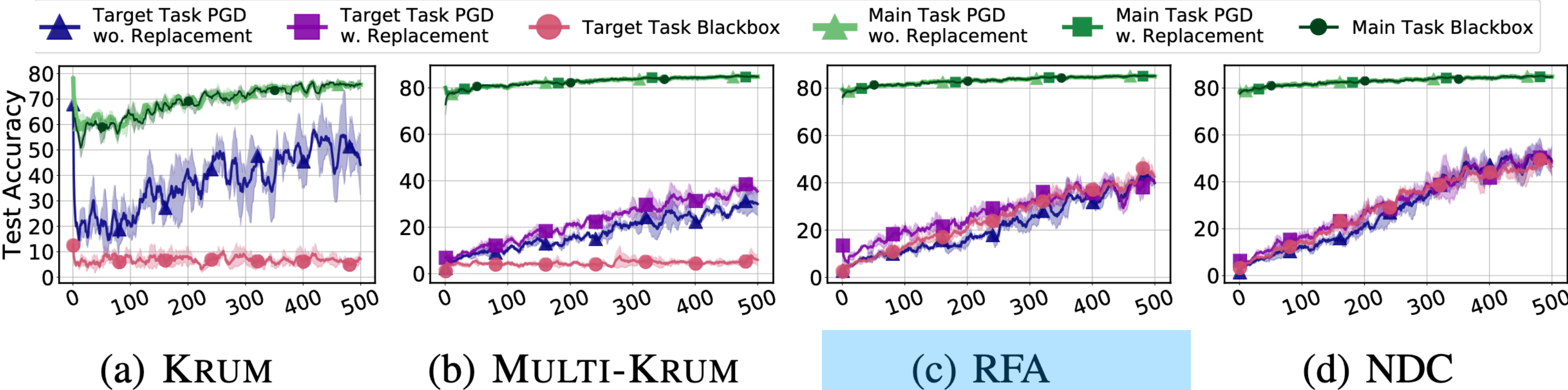
**Lê-Nguyen Hoang\***  
Calicarpa, Tournesol

**[AISTATS 2023]**



# RFA is a strong baseline

Wang et al.  
(NeurIPS 2020)



Li et al. (IEEE Trans. Industrial Informatics 2023)

See also Sejwalkar et al. (IEEE Security & Privacy 2022), Jin & Li (Medical Image Analysis 2023), Li et al. (IEEE Trans. Big Data 2023), ...

# Algorithmic advances based on RFA

Park et al. (NeurIPS 2021): RFA + Entropy-based reweighting

Karimireddy et al. (ICLR 2022): RFA + Bucketing

Li et al. (IEEE Trans. Ind. Inform. 2023): RFA + adaptive weighting

Allouah et al. (AISTATS 2023): RFA + nearest neighbors

⋮



# Fast and differentiable geometric median

```
import torch
from geom_median.torch import compute_geometric_median # PyTorch API
# from geom_median.numpy import compute_geometric_median # NumPy API

points = [torch.rand(d) for _ in range(n)] # list of n tensors of shape (d,)
# The shape of each tensor is the same and can be arbitrary (not necessarily 1-dimensional)
weights = torch.rand(n) # non-negative weights of shape (n,)
out = compute_geometric_median(points, weights)
# Access the median via `out.median`, which has the same shape as the points, i.e., (d,)
```

Install: **`pip install geom-median`**

Documentation: [github.com/krishnap25/geom-median](https://github.com/krishnap25/geom-median)



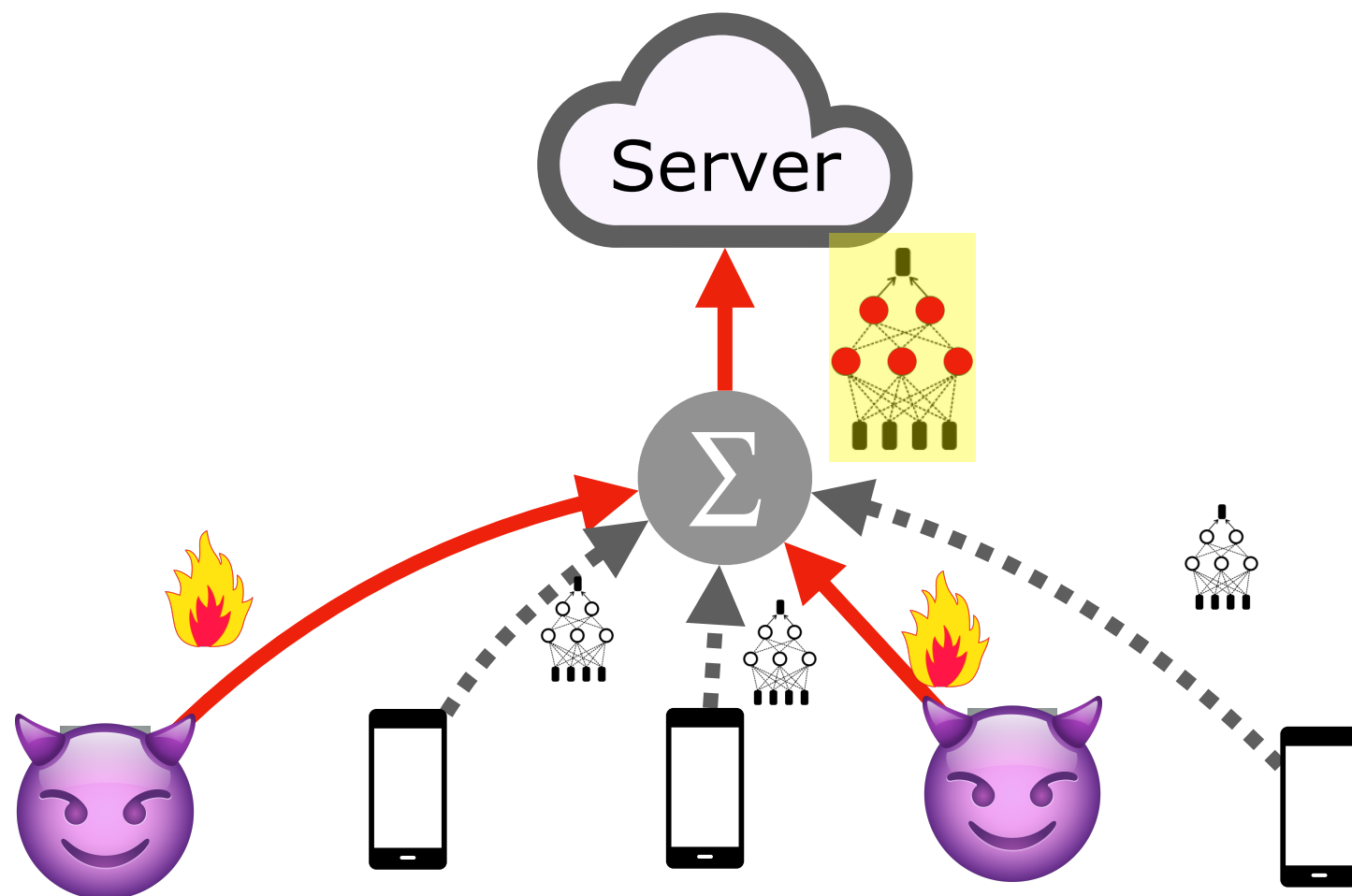
GitHub Link

# Summary

***Paper:***



Federated learning is  
***not robust*** to  
poisoned updates



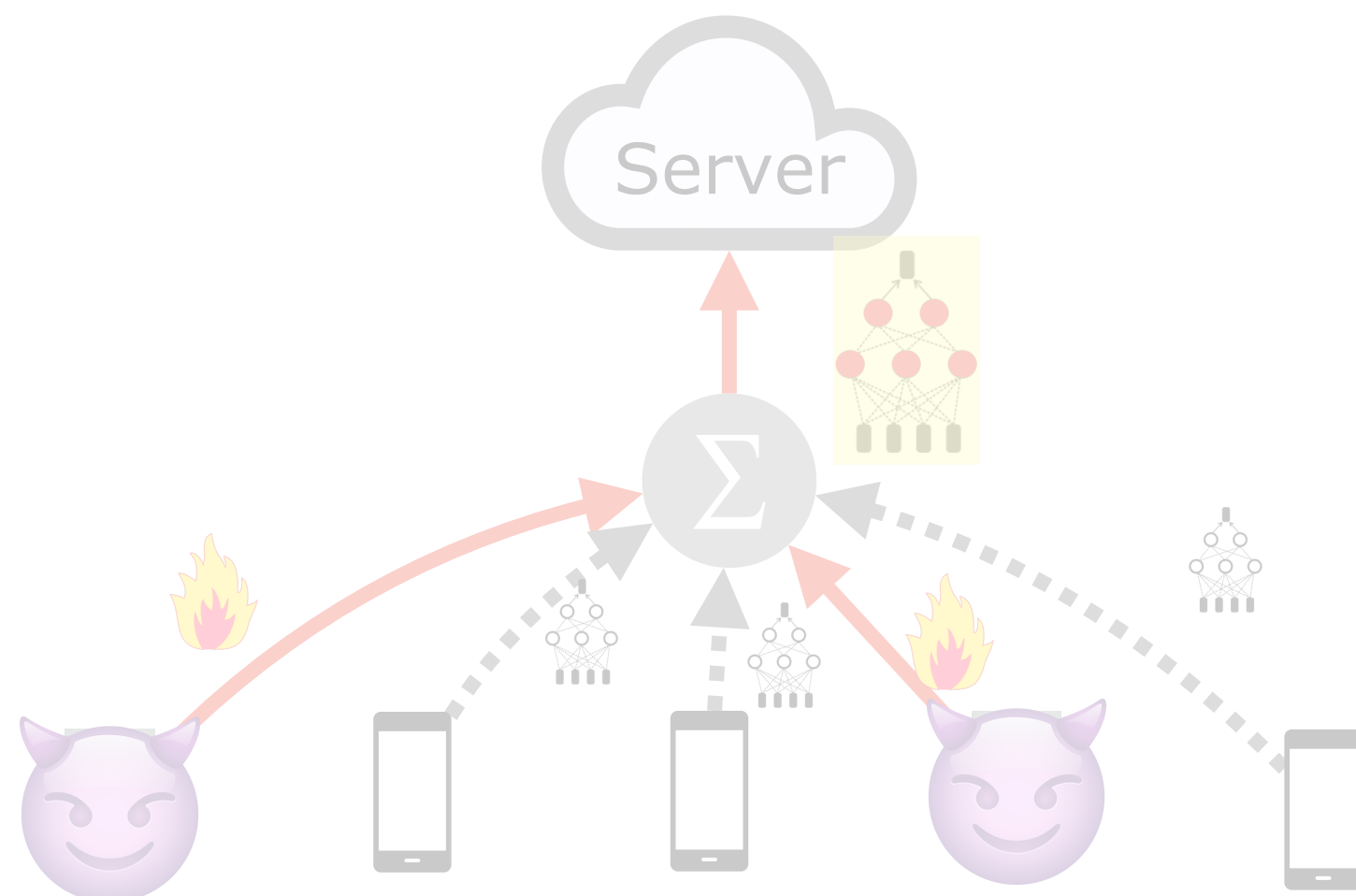


**Paper:**

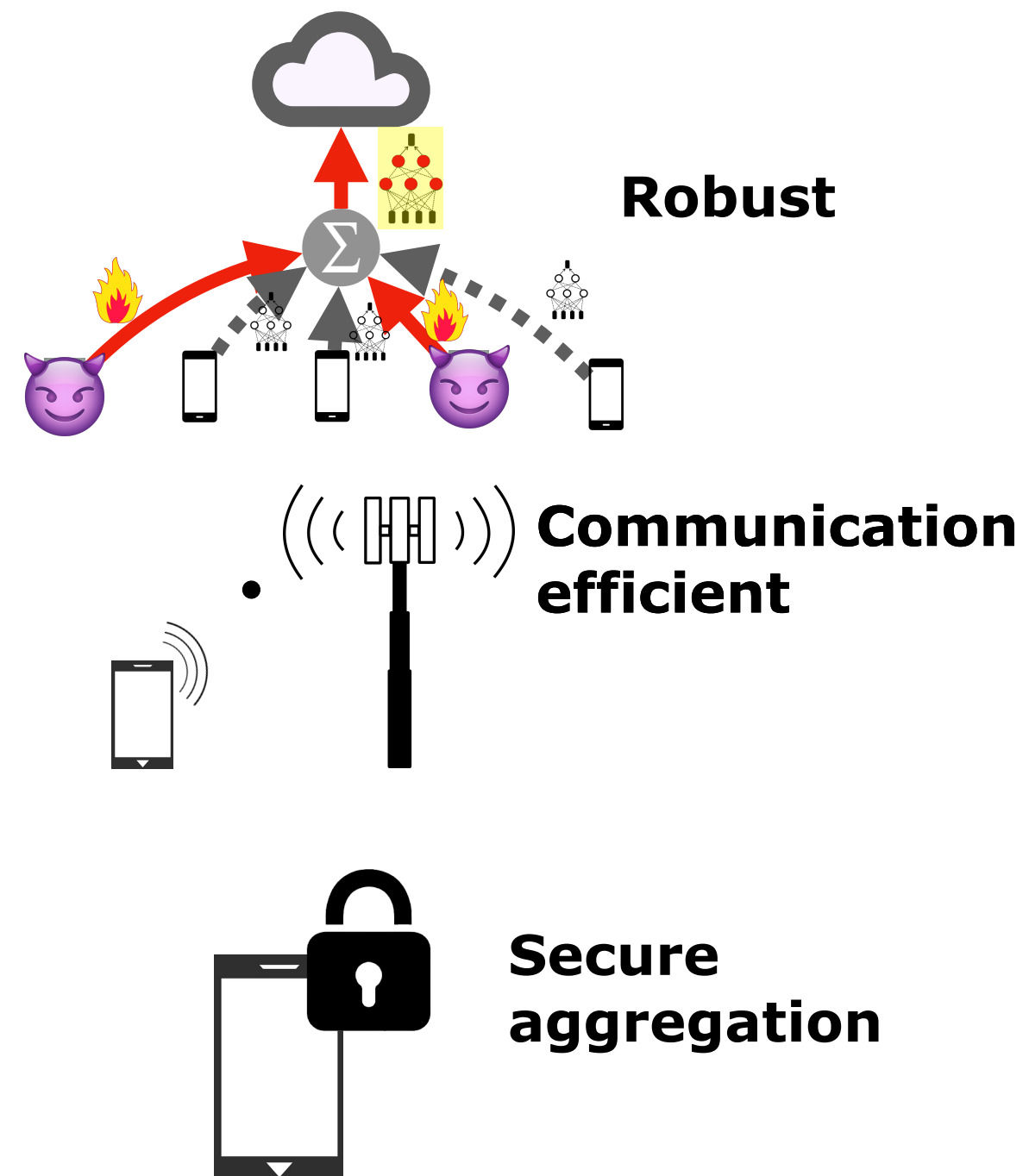


# Summary

Federated learning is ***not robust*** to poisoned updates



$$\text{GM} = \arg \min_z \sum_{i=1}^m \|z - w_i\|_2$$

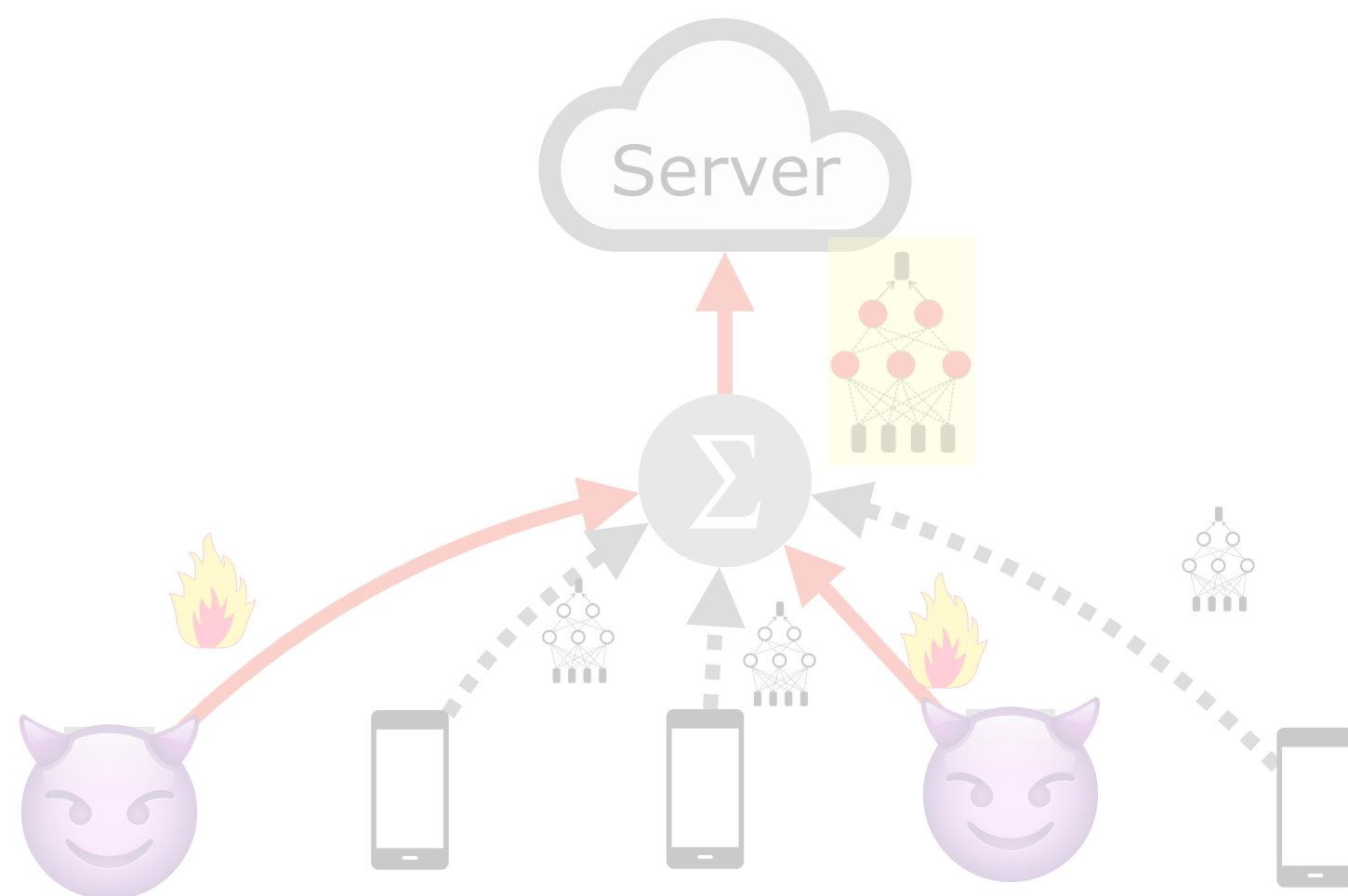


# Summary

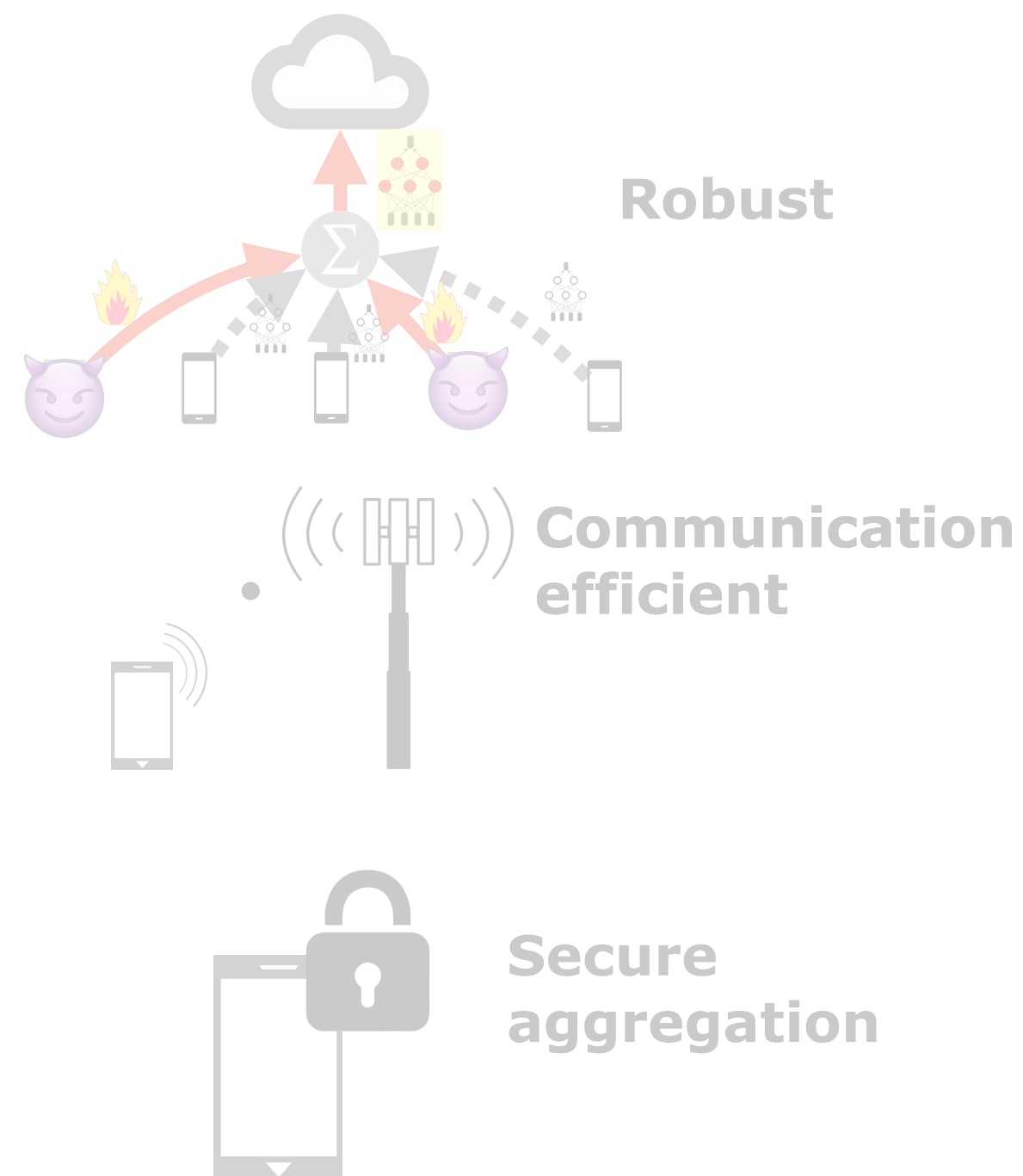
**Paper:**



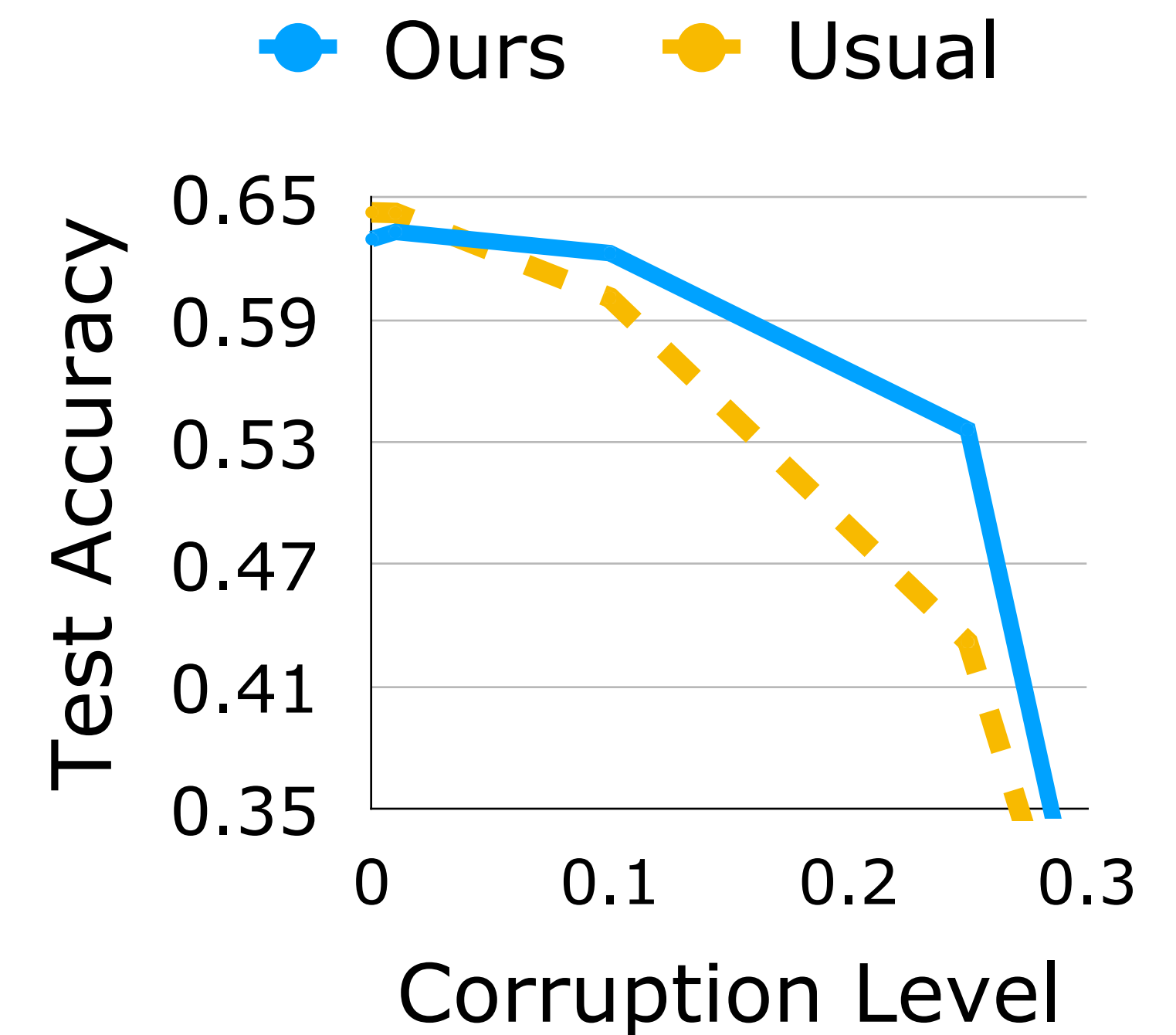
Federated learning is ***not robust*** to poisoned updates



$$GM = \arg \min_z \sum_{i=1}^m \|z - w_i\|_2$$

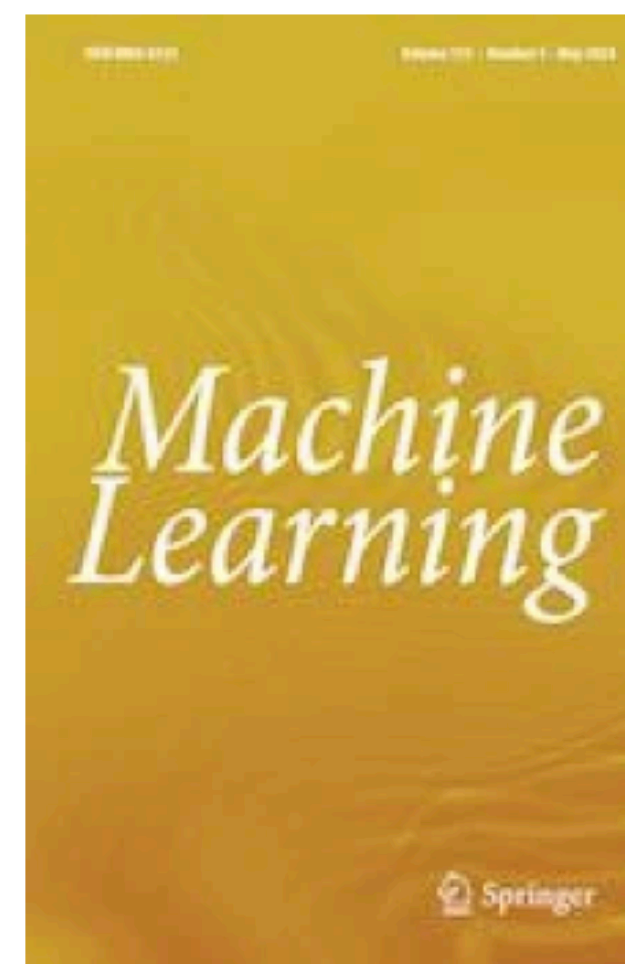


Our approach gives **greater robustness**





# Heterogeneity, fairness, equity with differential privacy in federated learning



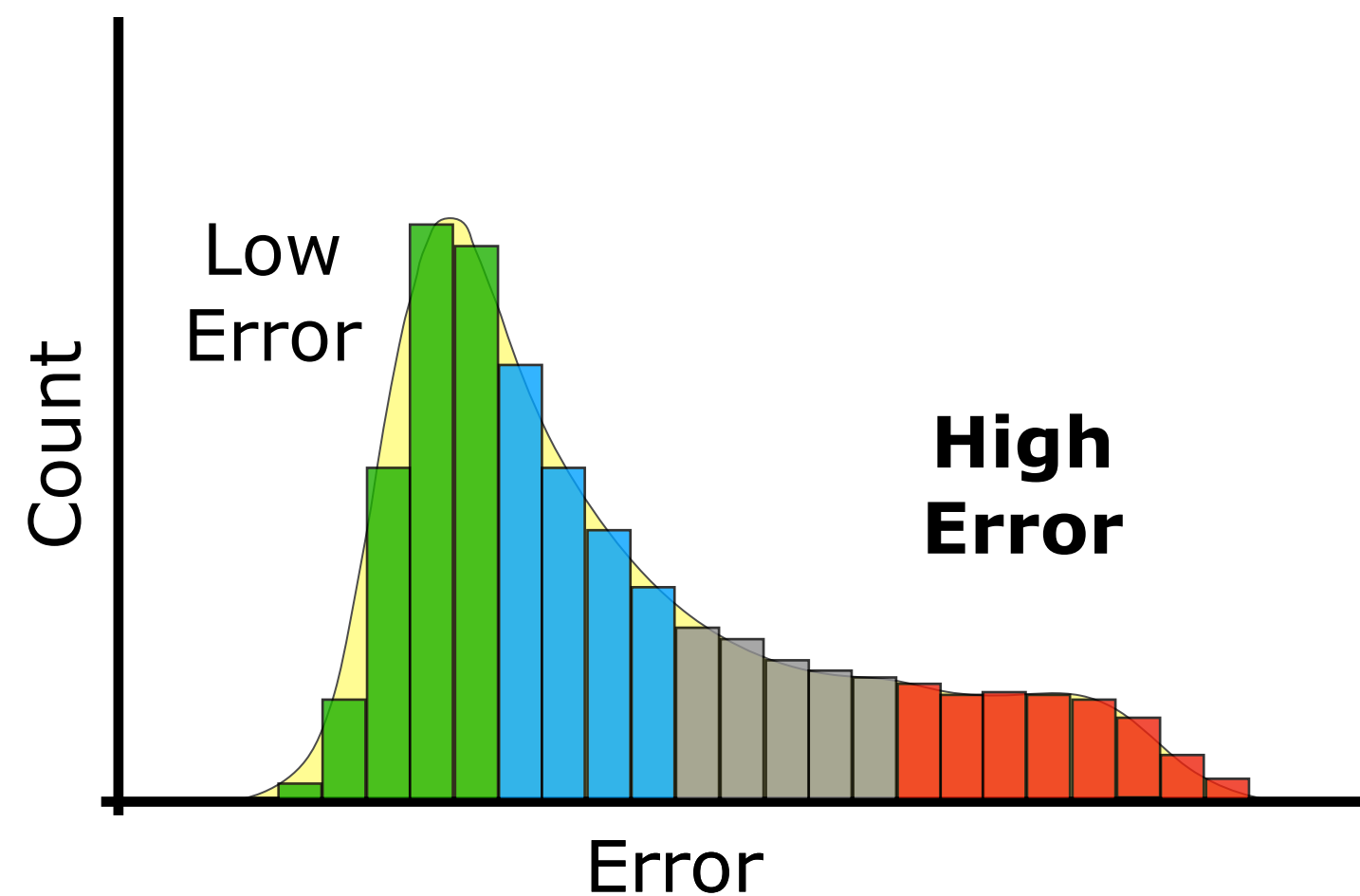
**Volume 113, Issue 5**

May 2024

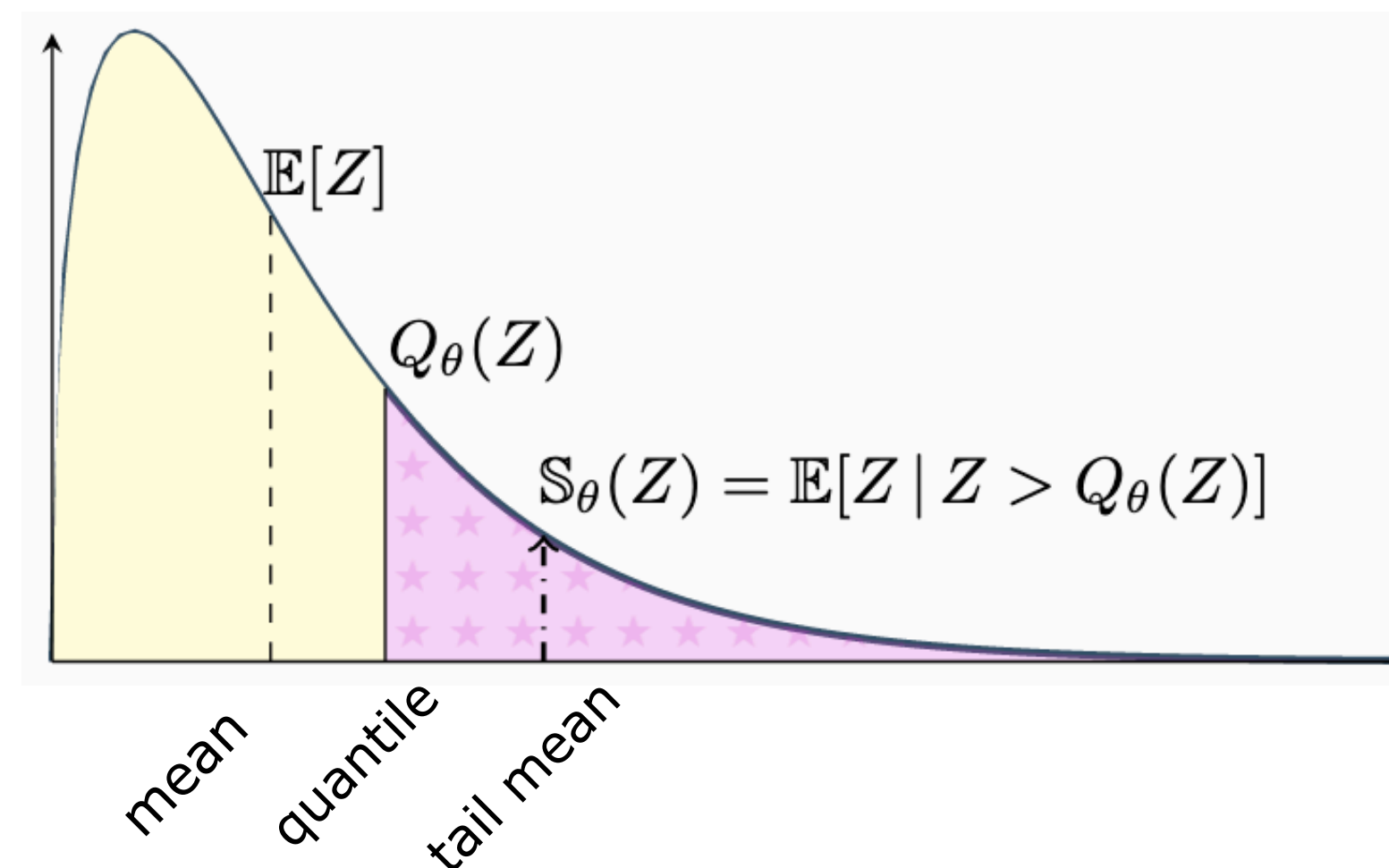
***Paper:***



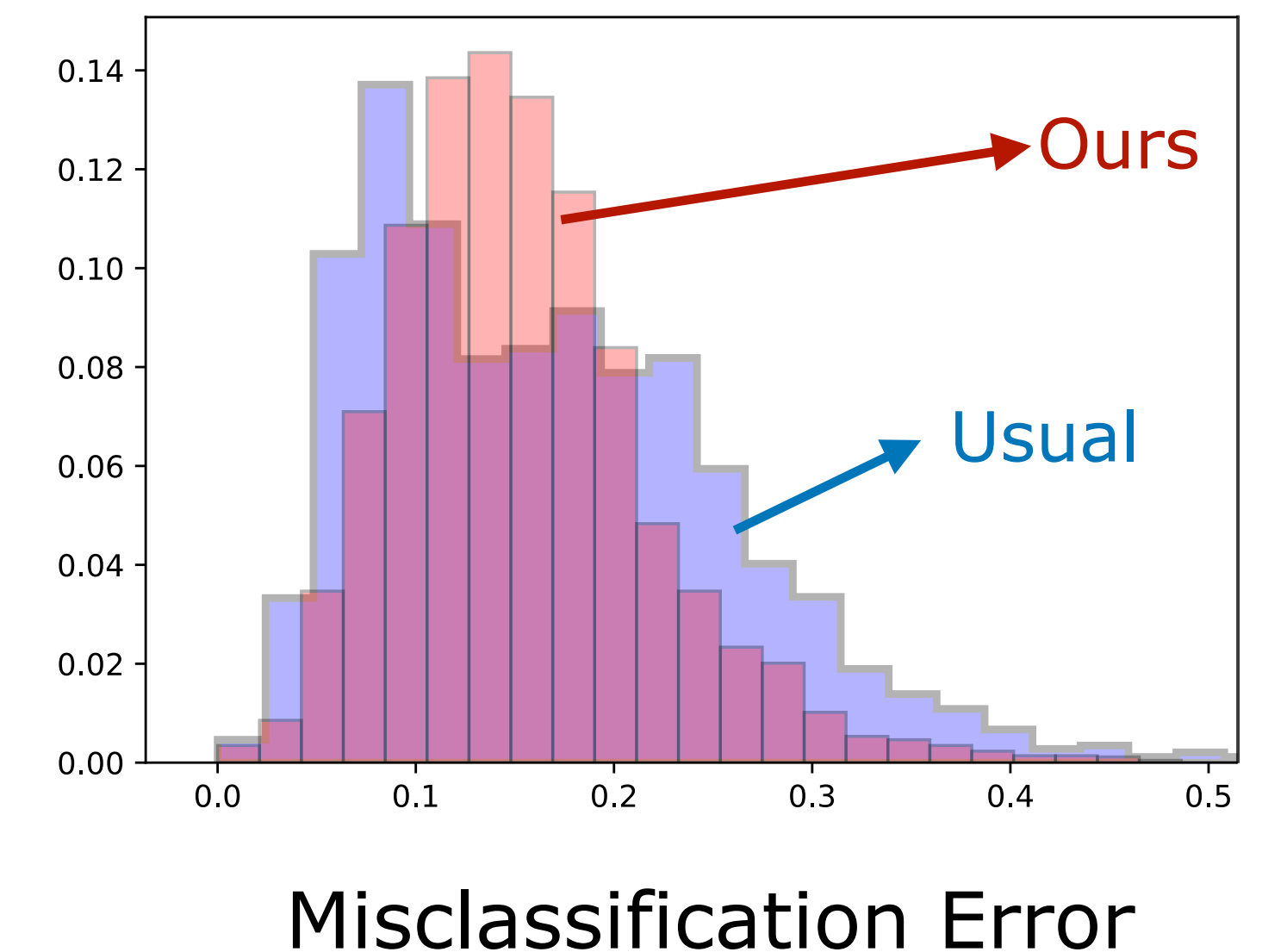
Distribution shift  $\Rightarrow$   
large tail errors



Minimize the tail  
error directly



We reduce tail error  
+ support differential  
privacy



# Thank you!

***Paper:***



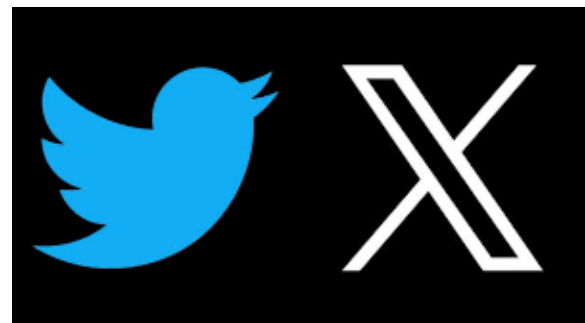
Software

```
pip install geom-median
```



Code

<https://github.com/krishnap25/tRFA>



@KrishnaPillutla